



7-th School-Conference on  
**Atomistic Simulation of Functional  
Materials**

(ASFM-2018 Fall) Moscow, September 12–13, 2018

*Ab initio* simulation of formation  
of the **conducting solid hydrogen**  
at high pressures

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# Outline

1. Metallic hydrogen. Introduction.

2. Calculation method.

3. Validation.

4. Molecular phase at high pressure.

5. Conducting phase.

6. Conclusions.

## § 2. V.L. Ginzburg's list of

'Especially Important and Interesting Problems'

1. Controlled thermonuclear fusion
2. High and room-temperature superconductivity
3. Metallic hydrogen.

4.

5.

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totally 30 problems in  
*macrophysics, microphysics and astrophysics*

Wigner, E.; Huntington, H.B.

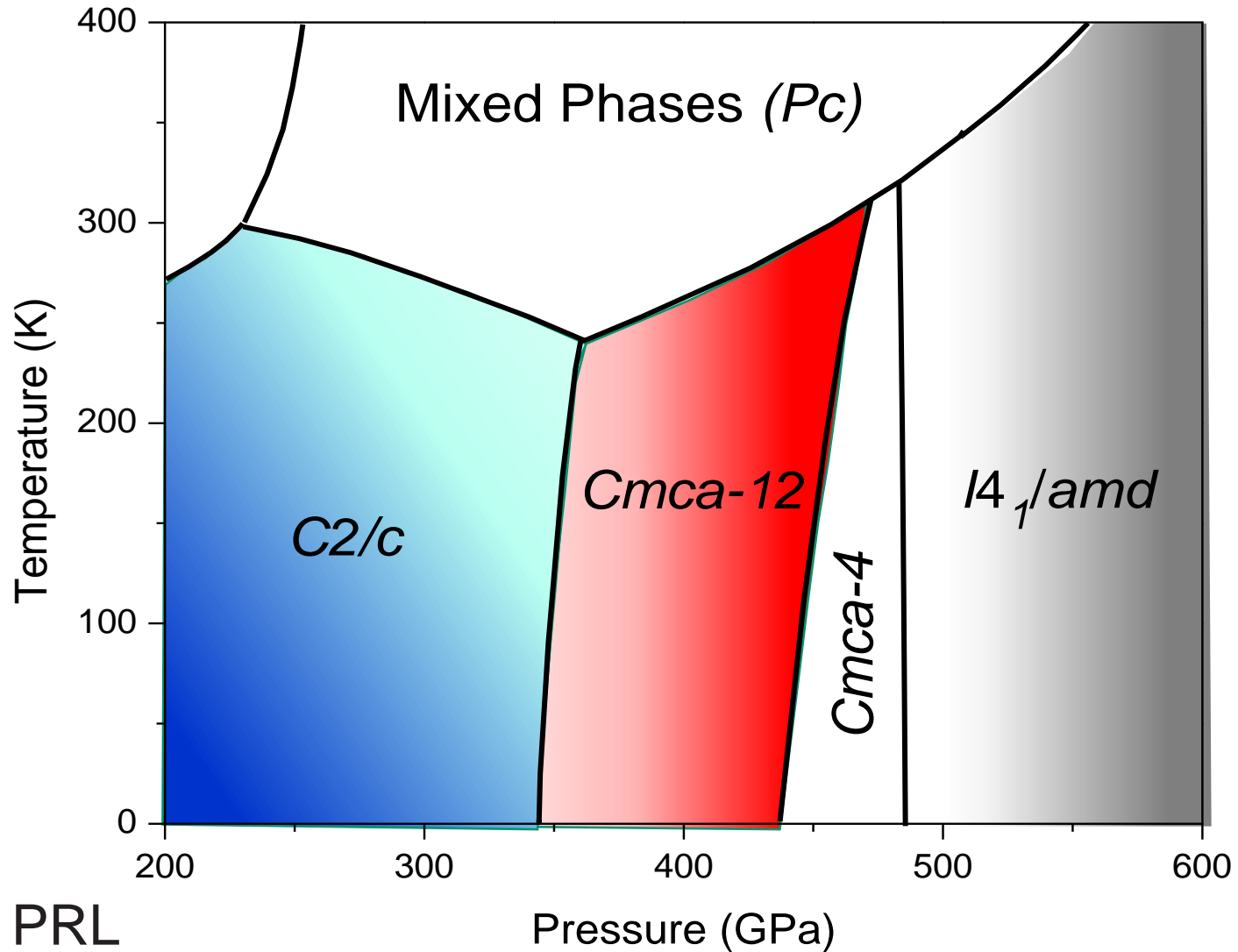
**On the possibility  
of a metallic modification  
of hydrogen**

Journal of Chemical Physics. 3, no. 12. 764 (1935)

0.25 Mbar

$T = 0 \text{ K}$

# Theoretical phase diagram of solid hydrogen



# High temperature superconductor

N. W. Ashcroft, Metallic Hydrogen: A High Temperature Superconductor?  
*Phys. Rev. Lett.* 21, 1748–1749 (1968).

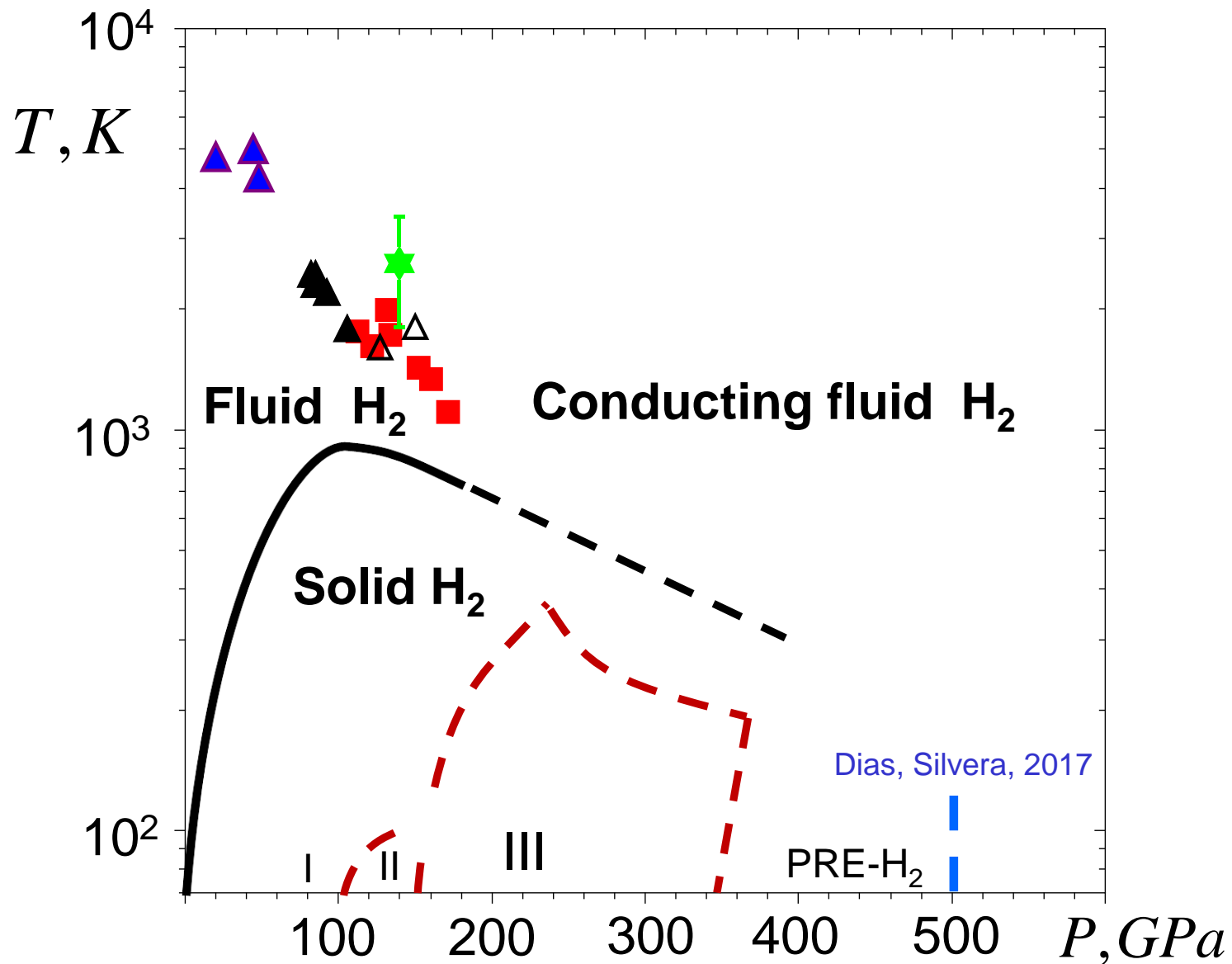
J. M. McMahon, D. M. Ceperley, High-temperature superconductivity in atomic metallic hydrogen. *Phys. Rev. B* 84, 144515 (2011).

M. Borinaga, I. Errea, M. Calandra, F. Mauri, A. Bergara, Anharmonic effects in atomic hydrogen: Superconductivity and lattice dynamical stability. *Phys. Rev. B* 93, 174308 (2016).

# Metastability at normal pressure

E. G. Brovman, Y. Kagan, A. Kholas, Structure of Metallic Hydrogen at Zero Pressure. *Sov. Phys. JETP* 34, 1300–1315 (1972).

# Dense H<sub>2</sub> phase diagram



## 2. Calculation method.



# METHOD

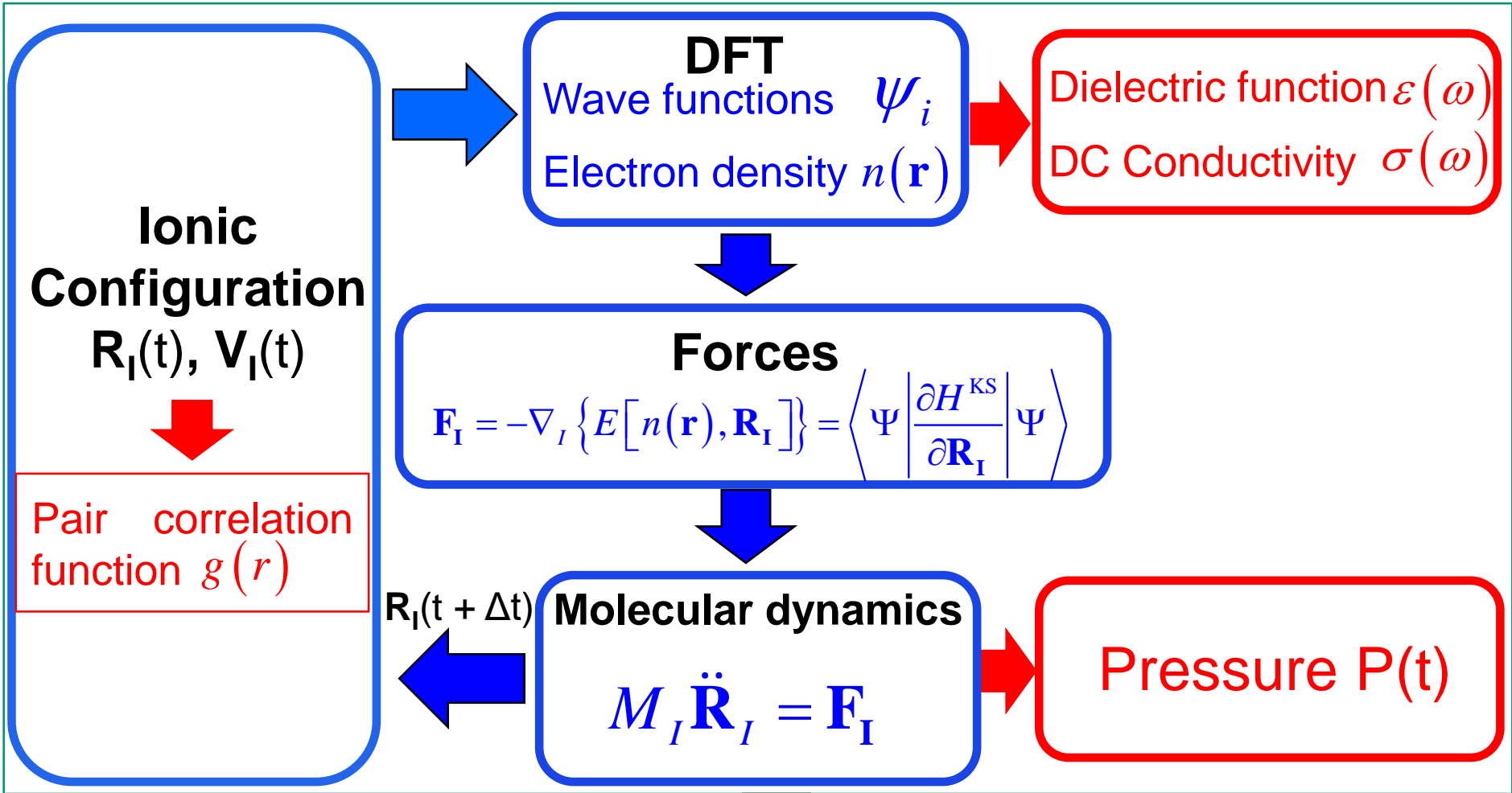
1. Calculation of the ions trajectories determined by the solution of the Newton's equations of motion with forces obtained within the DFT

2. Equilibration of the system

3. Calculation of parameters of the system:  
Proton-proton pair correlation function  $g(r)$

Pressure

Conductivity



**Averaging over configurations**  
**Pair correlation function, pressure, DC conductivity**

# Finite temperature Density Functional Theory

$$\Omega[n(\mathbf{r})] = \sum_i f_i \int \psi_i^* \left( -\frac{\hbar^2}{2m} \Delta \right) \psi_i d\mathbf{r} + \int V_{ext}(\mathbf{r}) n(\mathbf{r}) d\mathbf{r} + \frac{e^2}{2} \int \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}' d\mathbf{r} + E_{xc}[n(\mathbf{r})] - TS[n(\mathbf{r})]$$

$$n(\mathbf{r}) = \sum_i f_i(E_i, T) |\psi_i(\mathbf{r})|^2 \quad \langle \psi_i | \psi_j \rangle = \delta_{ij}$$

$$\left( -\frac{\hbar^2}{2m} \Delta + V_{ext}(\mathbf{r}) + V_H(\mathbf{r}) + V_{xc}(\mathbf{r}) \right) \psi_i(\mathbf{r}) = E_i \psi_i(\mathbf{r})$$

$$V_H = \frac{e^2}{2} \int \frac{n(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}' \quad V_{xc}(\mathbf{r}) = \frac{\delta E_{xc}[n(\mathbf{r})]}{\delta n(\mathbf{r})}$$


$$\psi_{n\mathbf{k}}(\mathbf{r}) = \sum_{\mathbf{G}}^{G_{\max}} C_{\mathbf{k}n\mathbf{G}} \cdot e^{i(\mathbf{k}+\mathbf{G})\mathbf{r}} \quad E_{cut} = \frac{\hbar^2 G_{\max}^2}{2m}$$

# Dielectric function

$$\epsilon = \epsilon^{(1)} + i \cdot \epsilon^{(2)}$$

Longitudinal expression for  
the imaginary part of dielectric function:

$$\epsilon^{(2)}(\omega) = \frac{1}{3} \cdot \frac{4\pi^2 e^2}{\Omega} \lim_{|\mathbf{q}| \rightarrow 0} \frac{1}{|\mathbf{q}|^2} \sum_{i,j,\alpha,\mathbf{k}} 2w_{\mathbf{k}} \cdot \left[ f(E_{i,\mathbf{k}+\mathbf{q}}) - f(E_{j,\mathbf{k}}) \right] \times$$
$$\times \left| \langle \Psi_{i,\mathbf{k}+\mathbf{e}_\alpha \mathbf{q}} | \Psi_{j,\mathbf{k}} \rangle \right|^2 \cdot \delta(E_{i,\mathbf{k}+\mathbf{q}} - E_{j,\mathbf{k}} - \hbar\omega)$$


$$\delta(x) \rightarrow \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

## Kramers-Kronig transformation

for the imaginary part of the dielectric function:

$$\varepsilon^{(1)}(\omega) = 1 + \frac{2}{\pi} P \int_0^{\infty} \frac{\varepsilon^{(2)}(\omega') \omega'}{\omega'^2 - (\omega + i\eta)^2} d\omega'$$

$$\sigma^{(1)}(\omega) = \varepsilon_0 \omega \varepsilon^{(2)}(\omega)$$

effective collisional frequency  $\eta = 0.01$  eV

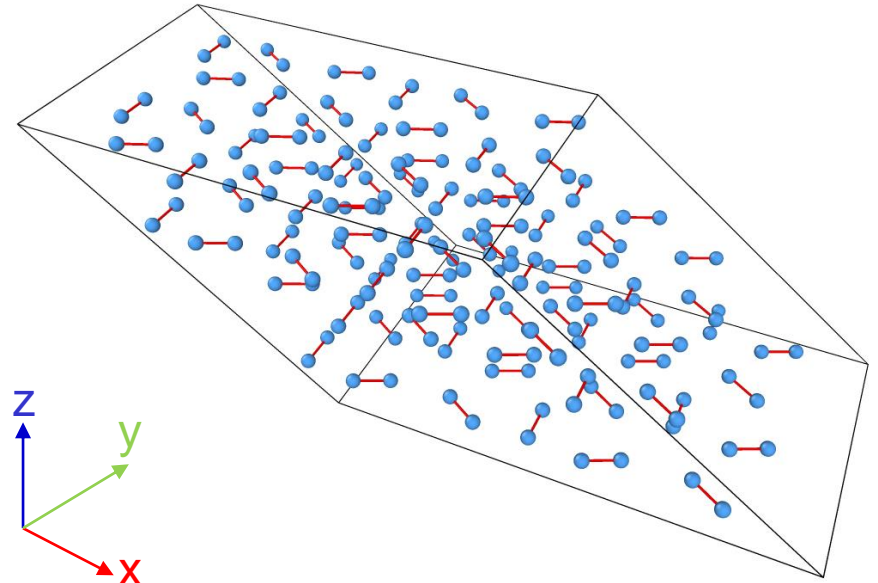
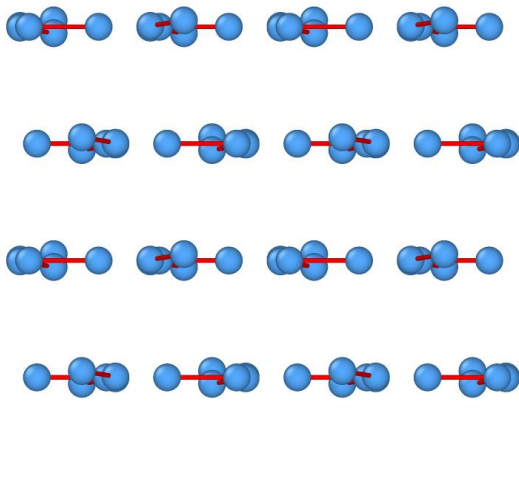
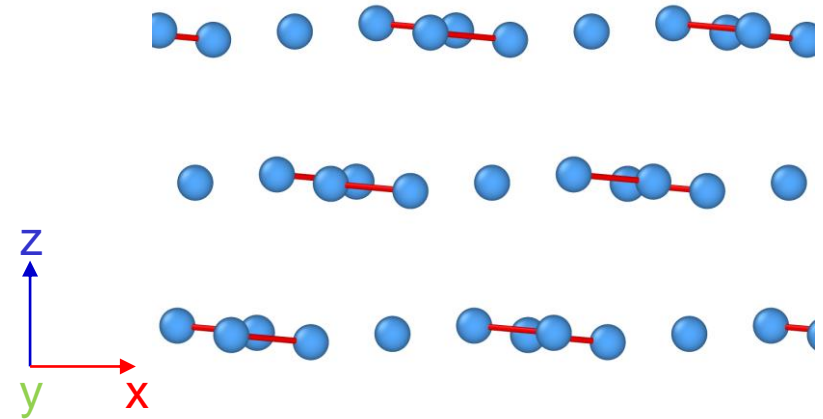
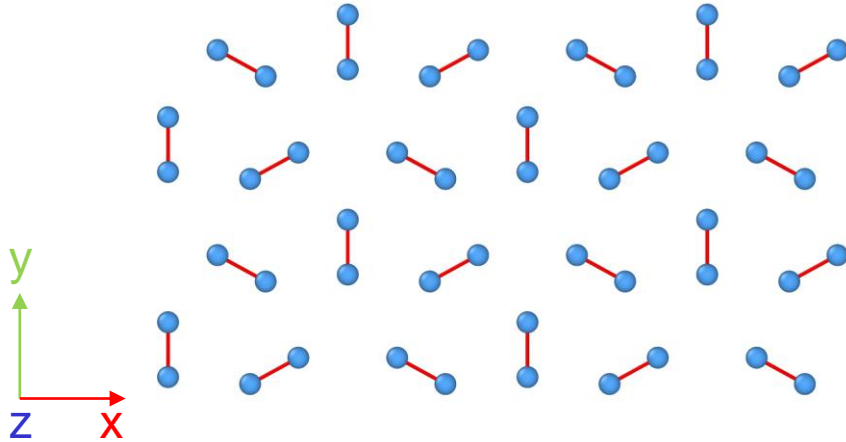
## 3. Validation.

# Initial configuration

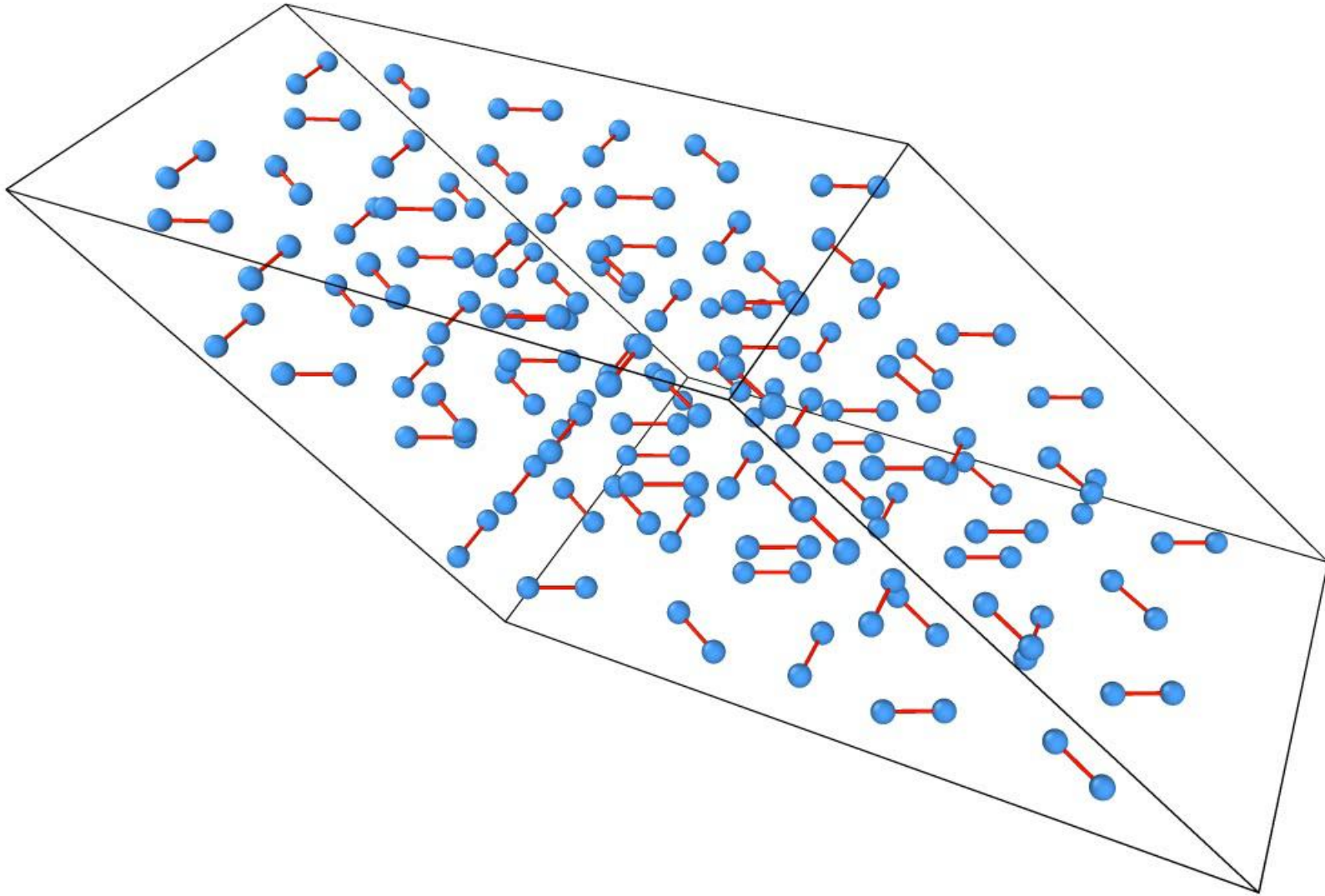
Monocline structure  
with **C2/c** space group  
24 atoms in the unit cell

Density: 1.14 g/cm<sup>3</sup>  
Pressure: 302 GPa  
Number of particles: 192

C.J. Pickard, R.J. Needs // Nature Phys. **3**, 473 (2007)



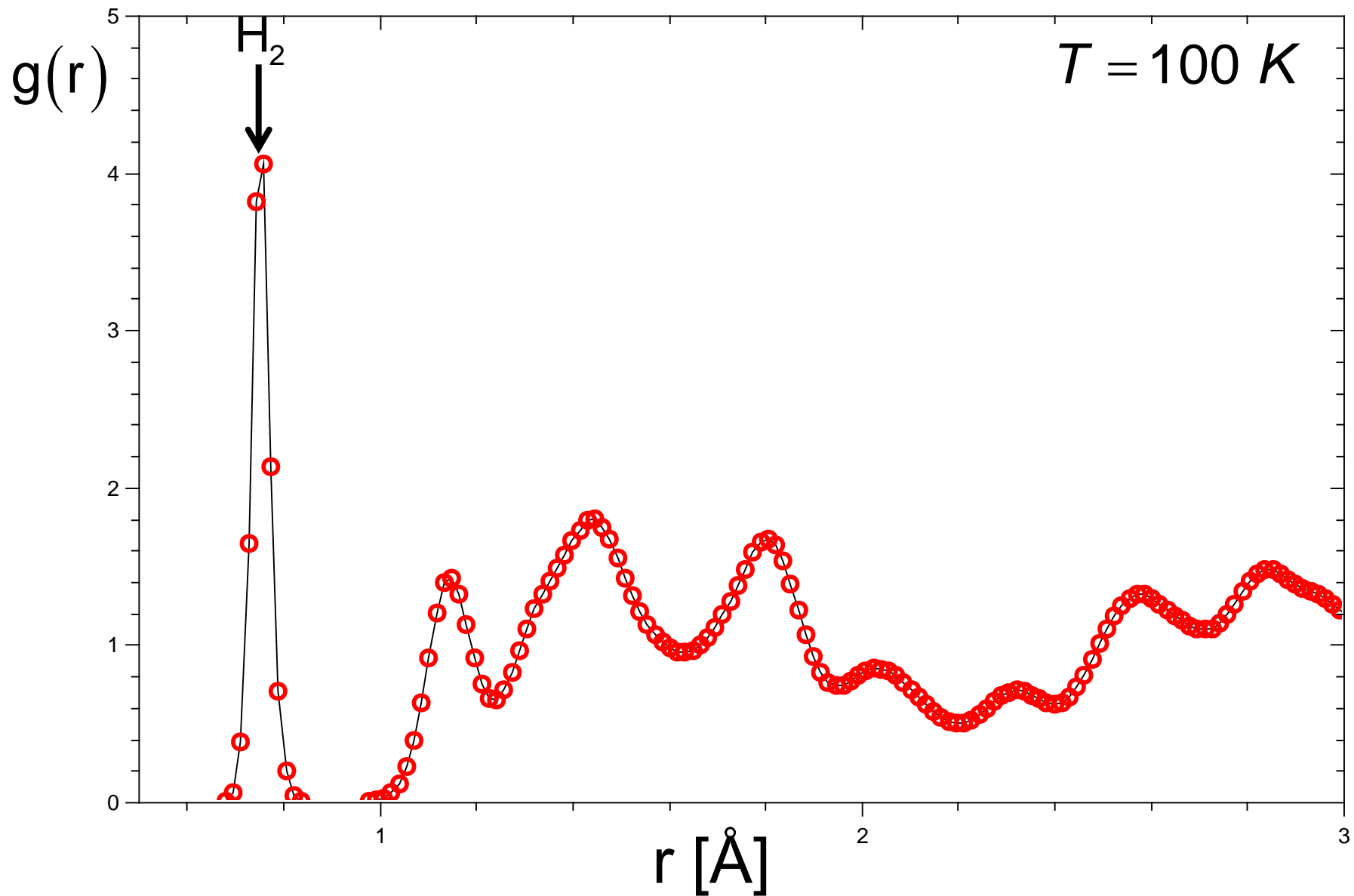
# MD at initial density



Conductivity:  $0.58 \text{ (Ohm}\cdot\text{cm)}^{-1}$

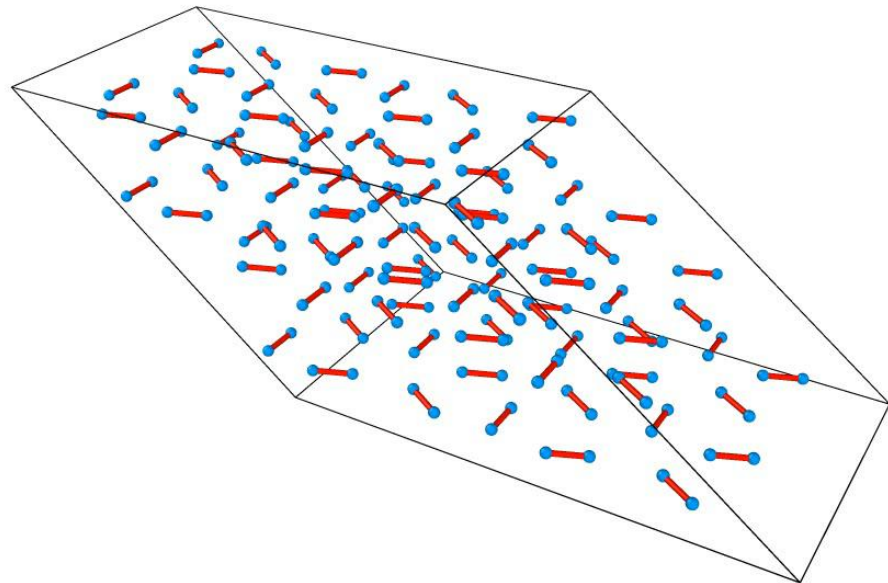
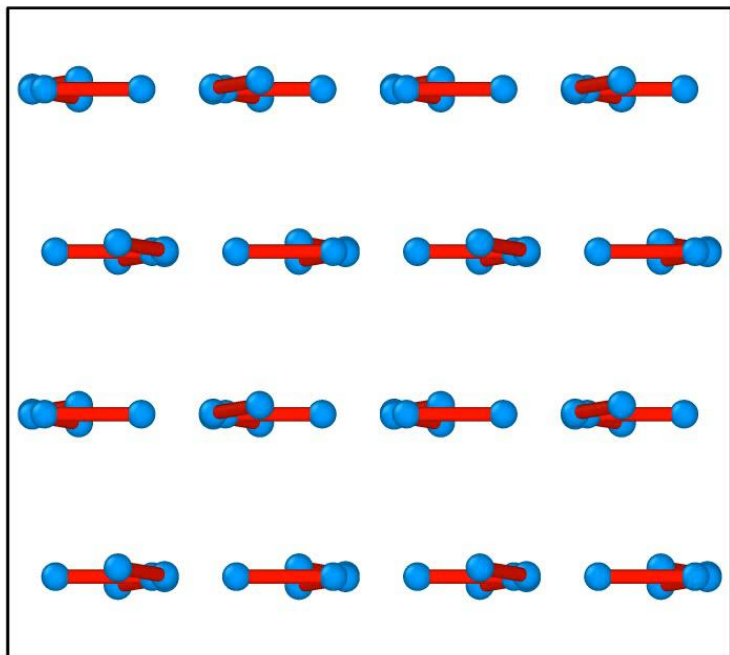
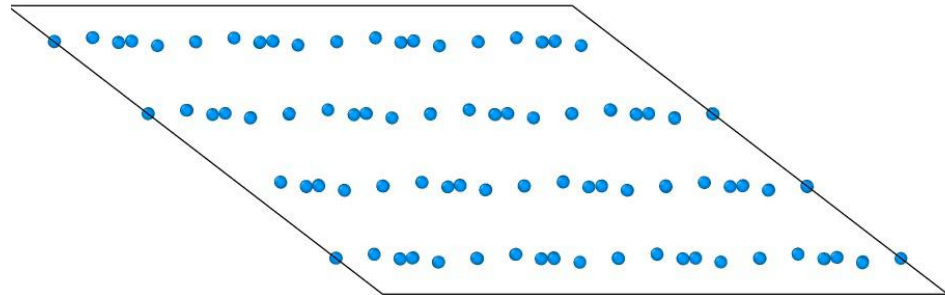
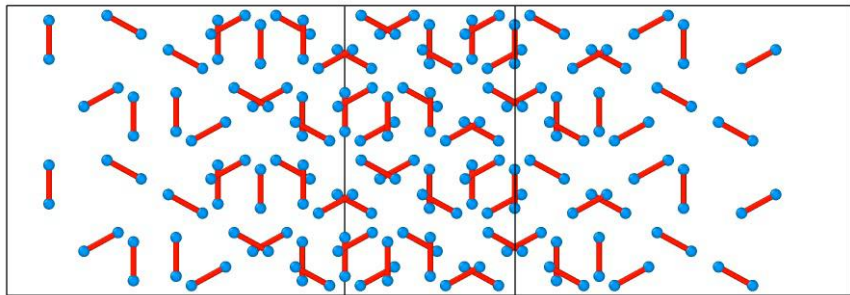


# Pair correlation function at pressure 302 GPa



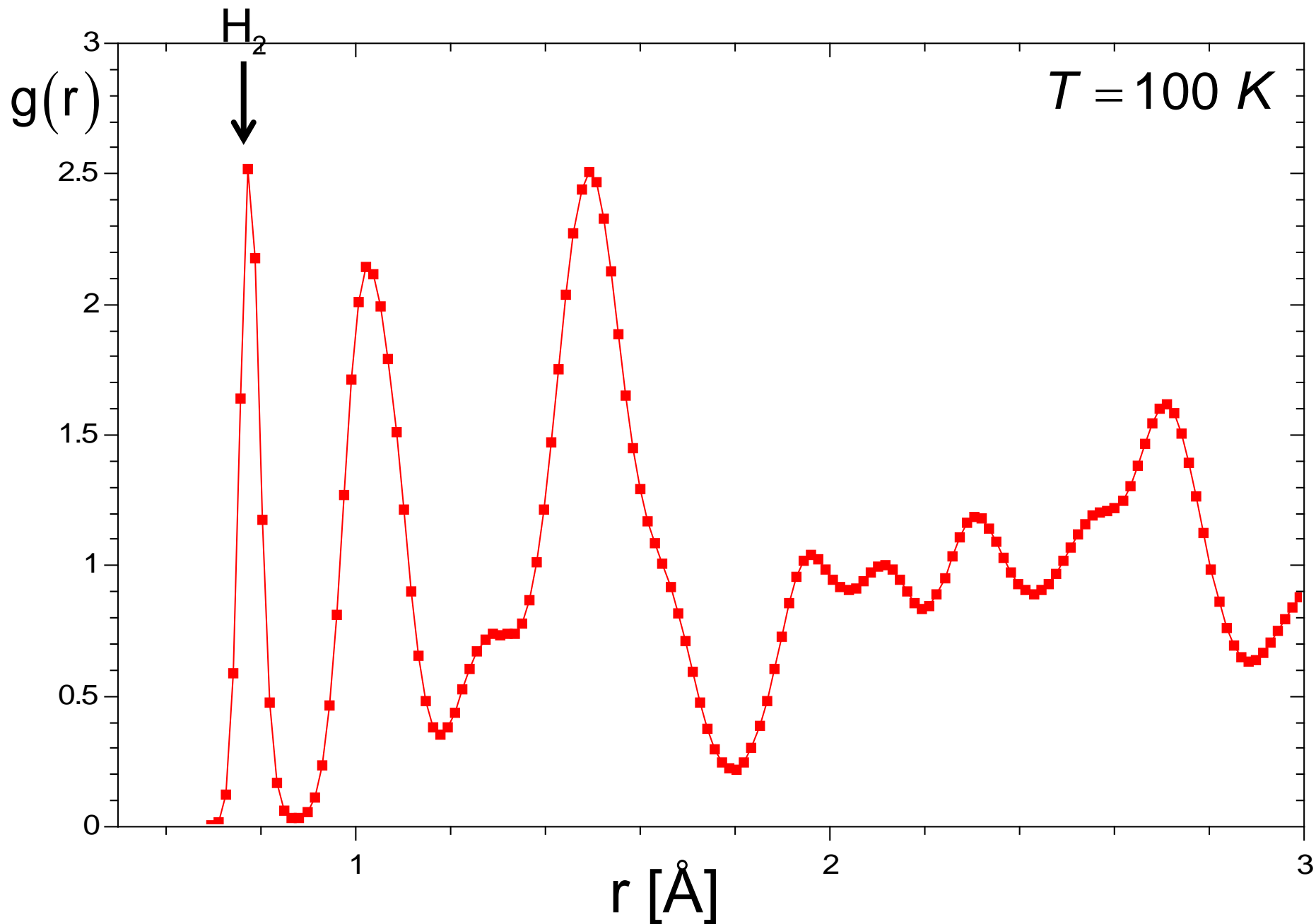
## 4. Molecular phase at high pressure

# MD at density $1.55 \text{ g/cm}^3$



Conductivity at density  $1.55 \text{ g/cm}^3$ :  $5.32 \text{ (Ohm}\cdot\text{cm)}^{-1}$

# Pair correlation function at pressure 585 GPa



## Space group: *Cmca*

$a=1.450$     $b=2.567$     $c=2.482$    H1   0.0000   0.1132   0.1262  
 $\alpha=90.00$     $\beta=90.00$     $\gamma=90.00$

**C.J. Pickard, R.J. Needs // Nature Phys. 3, 473 (2007)**

# Comments

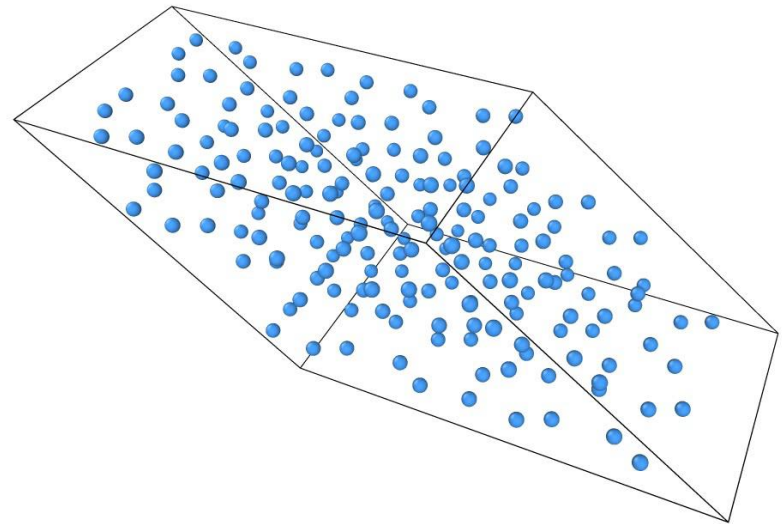
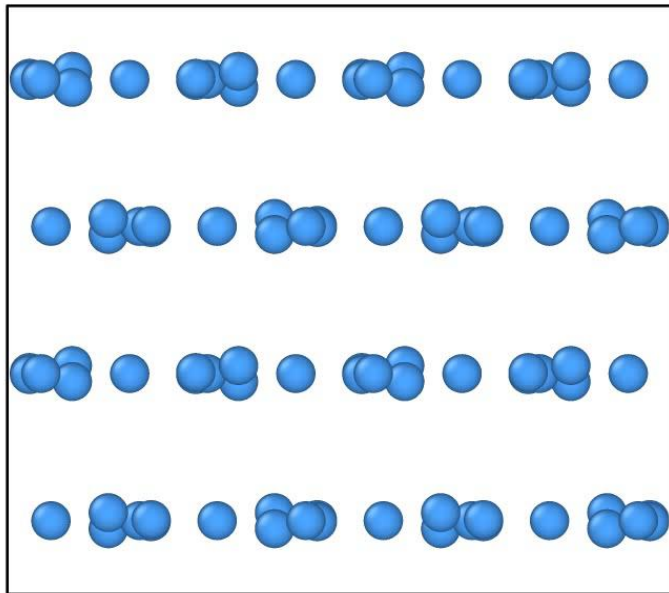
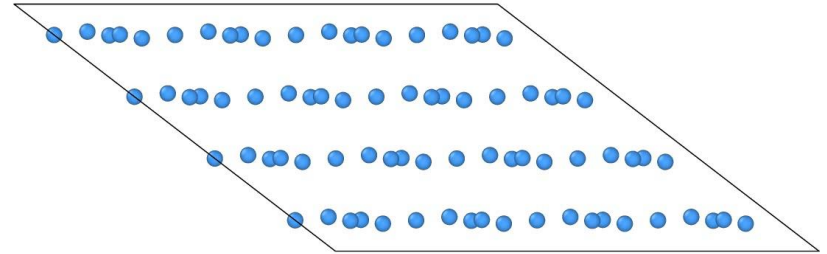
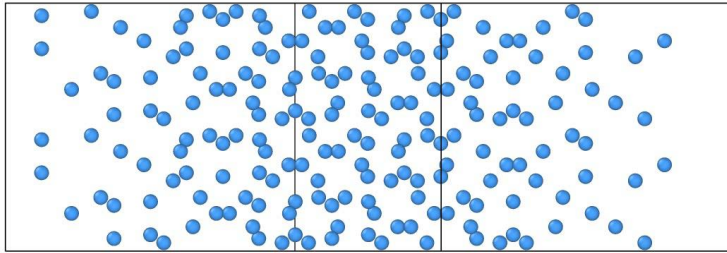
We do nothing to create structures.

We use the structure with C2/c space group as  
are initial conditions in each case.

Formation of one or another molecular structure  
is a result of equilibration  
at the quantum molecular dynamics simulation

## 5. Conducting phase

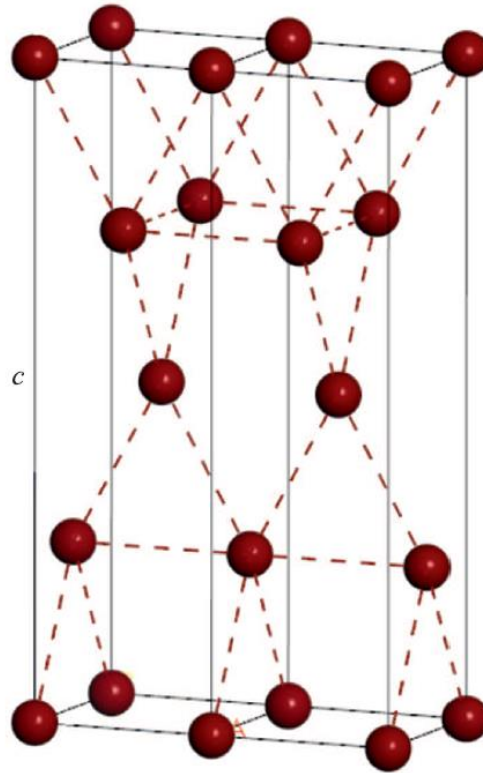
# MD at density 1.563 g/cm<sup>3</sup>



Conductivity at density 1.563 g/cm<sup>3</sup>: 85300 (Ohm·cm)<sup>-1</sup>



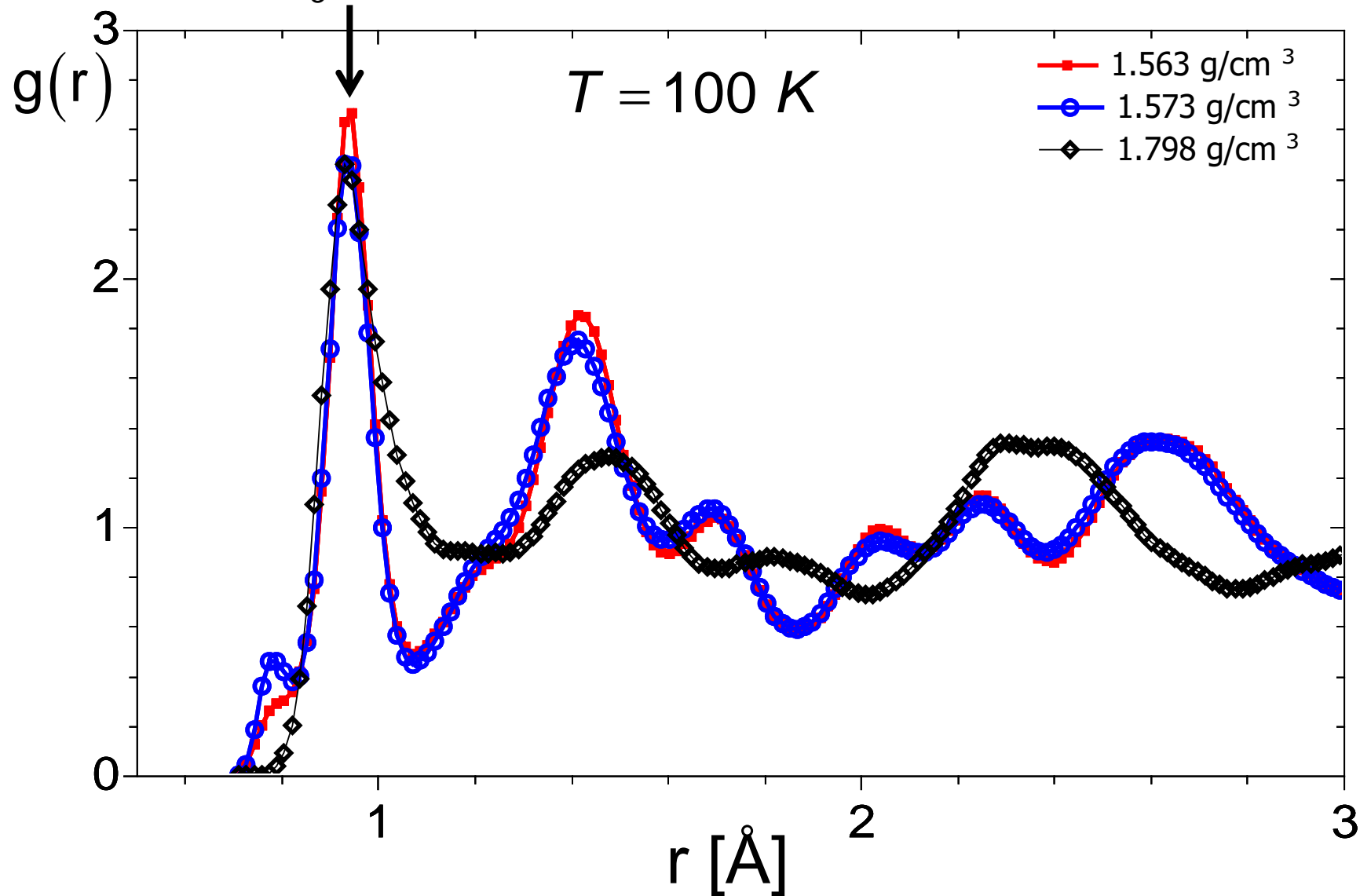
## Space group: $I4_1/amd$



Н.Н. Дегтяренко, Е.А. Мазур. Устойчивая структура  
металлического водорода при давлении 500 ГПа  
// Письма в ЖЭТФ, **104**, №5, с. 329 – 333 (2016)

# Pair correlation functions at pressure range 607 - 836 GPa

$\text{H}_3^+$   $r = 0.92 \text{ \AA}$



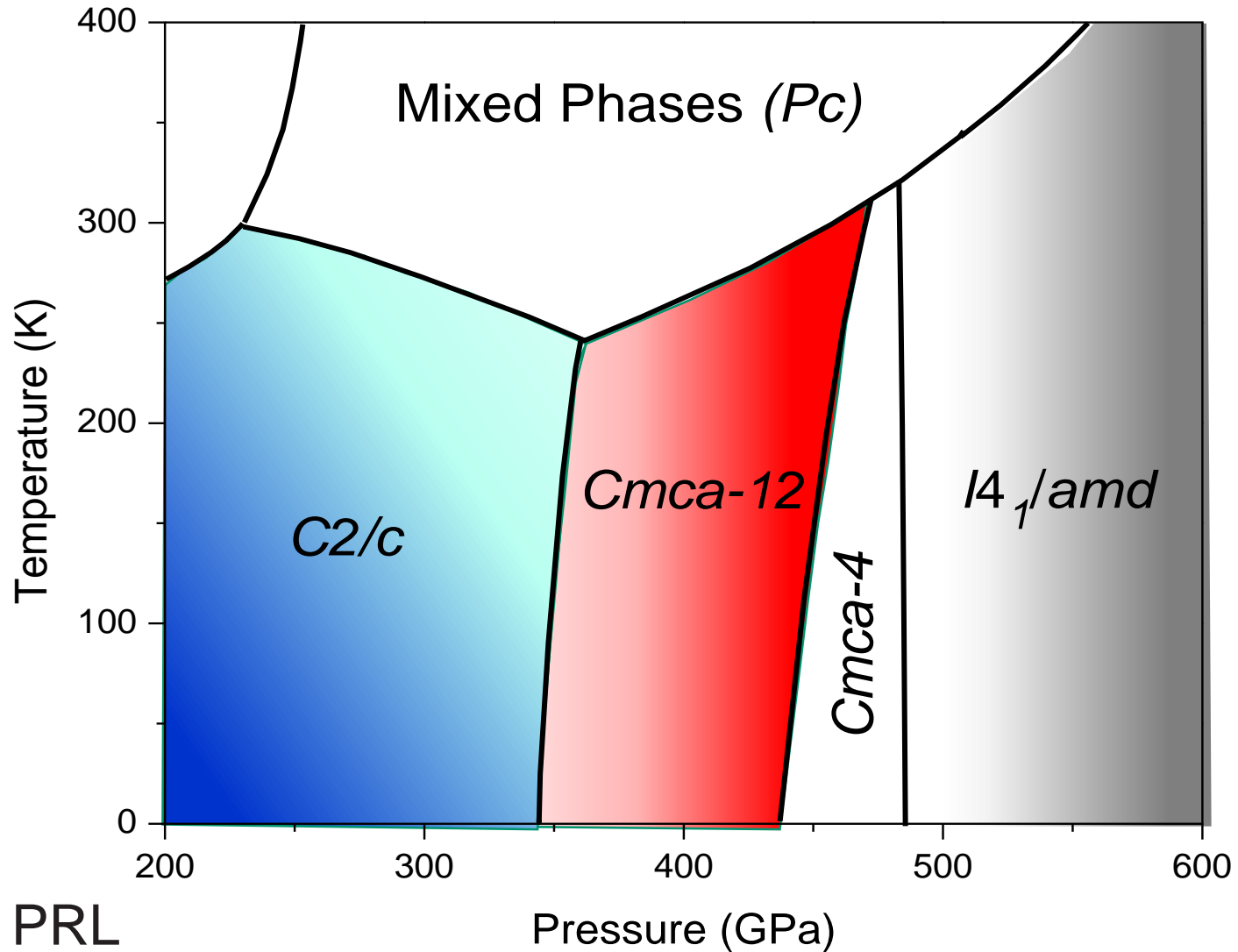
# Comments

We do nothing to create  
a new conducting structure.

We use the structure with C2/c space group as  
the initial condition in each case.

Formation of the conducting structure  
is a result of equilibration  
at the quantum molecular dynamics simulation

# Theoretical phase diagram of solid hydrogen



# 6. Conclusions

Formation of the intermediate **dielectric molecular phase** and the final **conducting phase** is observed during one and the same molecular dynamic trajectory

The PCF peak arises at a distance 0.92 Å at a density of 1.563 g/cm<sup>3</sup> and a pressure of 607 GPa, and it remains at the same distance up to the 836 GPa.

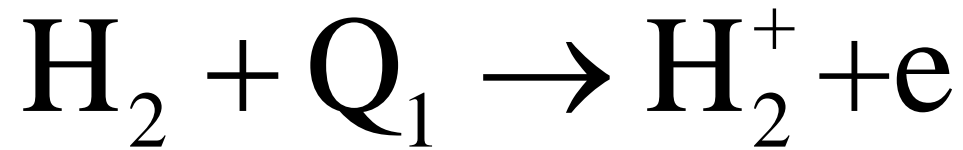
The distance 0.92 Å corresponds to the interatomic separation in H<sub>3</sub><sup>+</sup>

Wigner and Huntington and all subsequent authors considered only the proton lattice formation at the transition to the conducting state.



# Mechanism of the phase transition

## 1. Ionization



## 2. Subsequent equilibration

