

«7-th School-Conference on  
Atomistic Simulation of Functional Materials (ASFM 2018)»

# Predictive power of classical MD calculations for n-alkanes rheological properties

*N.D. Kondratyuk, G.E. Norman,*

*V.V. Pisarev and V.V. Stegailov*



JIHT RAS



NATIONAL RESEARCH  
UNIVERSITY



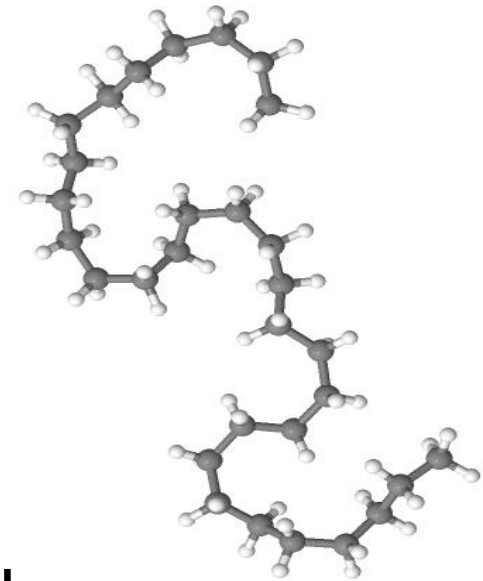
# План доклада



## 1. Самодиффузия в жидком *n*-триаконтане

Потенциалы взаимодействия  
(+ COMPASS)

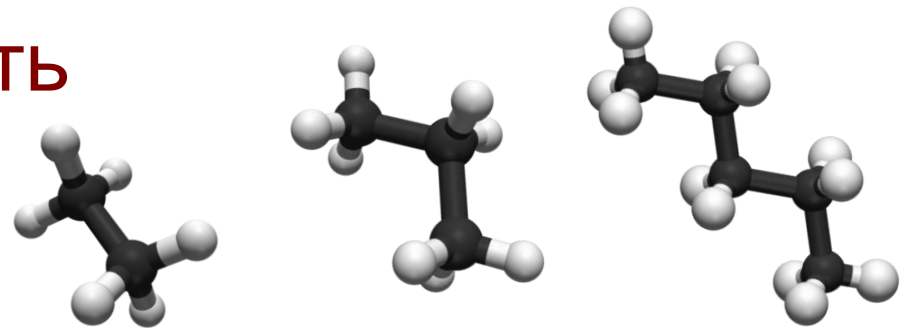
Предсказательная способность



*Молекула *n*-триаконтана*

## 2. Сдвиговая вязкость

Сходимость Г-К

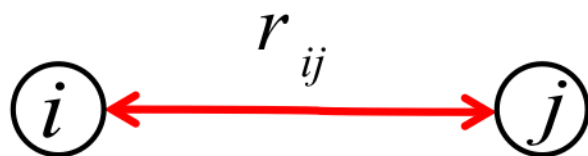


Неравновесная МД + эксперимент

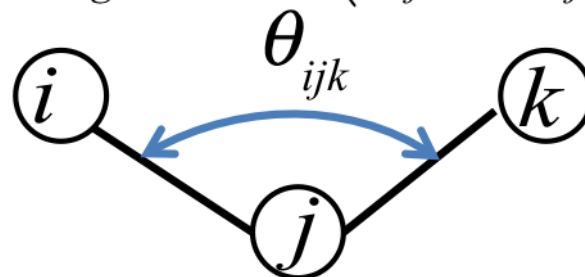
# Модель класса I

$$E = E_{bond} + E_{angle} + E_{dihedral} + E_{LJ} + E_{Coul}$$

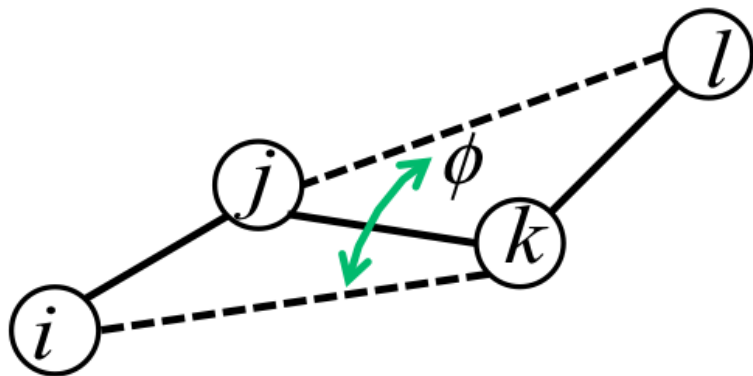
$$E_{bond} = K_b (r_{ij} - r_{ij}^0)^2$$



$$E_{angle} = K_a (\theta_{ijk} - \theta_{ijk}^0)^2$$

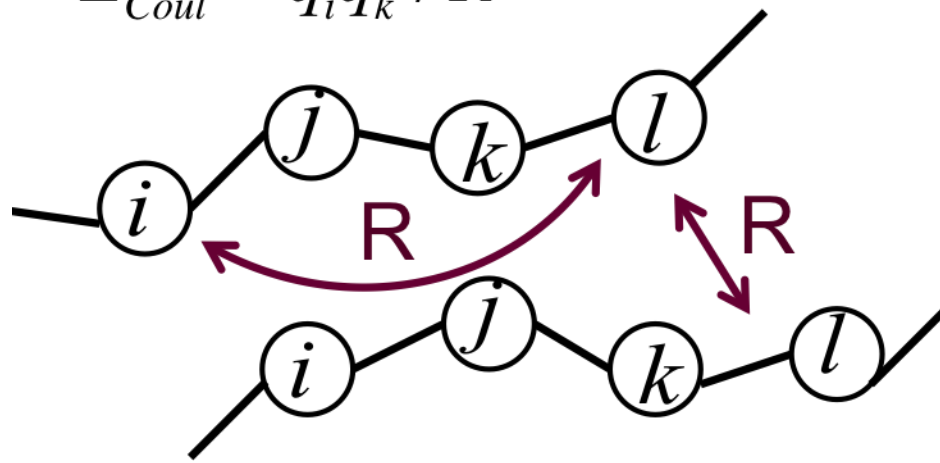


$$E_{dihedral} = \sum_i K_i [1 - \cos(i \cdot \phi)]$$



$$E_{LJ} = 4\epsilon [(\sigma / R)^{12} - (\sigma / R)^6]$$

$$E_{Coul} = q_i q_k / R$$



# Потенциалы

## TraPPE-UA (1996)

*Martin, M. G., Siepmann, J. I.* Transferable Potentials for Phase Equilibria. 1. United-Atom Description of n -Alkanes. // J. Phys. Chem. B. 1998. V. 102(14) P. 2569–2577

**1766 citations**

## OPLS-AA (1996)

Open Potential for Liquid Simulations – All-Atom

*Jorgensen W. L., Maxwell, D. S., Tirado-Rives J.* Development and Testing of the OPLS All-Atom Force Field on Conformational Energetics and Properties of Organic Liquids // J. Am. Chem. Soc. 1996. V. 118(45). P. 11225–11236.

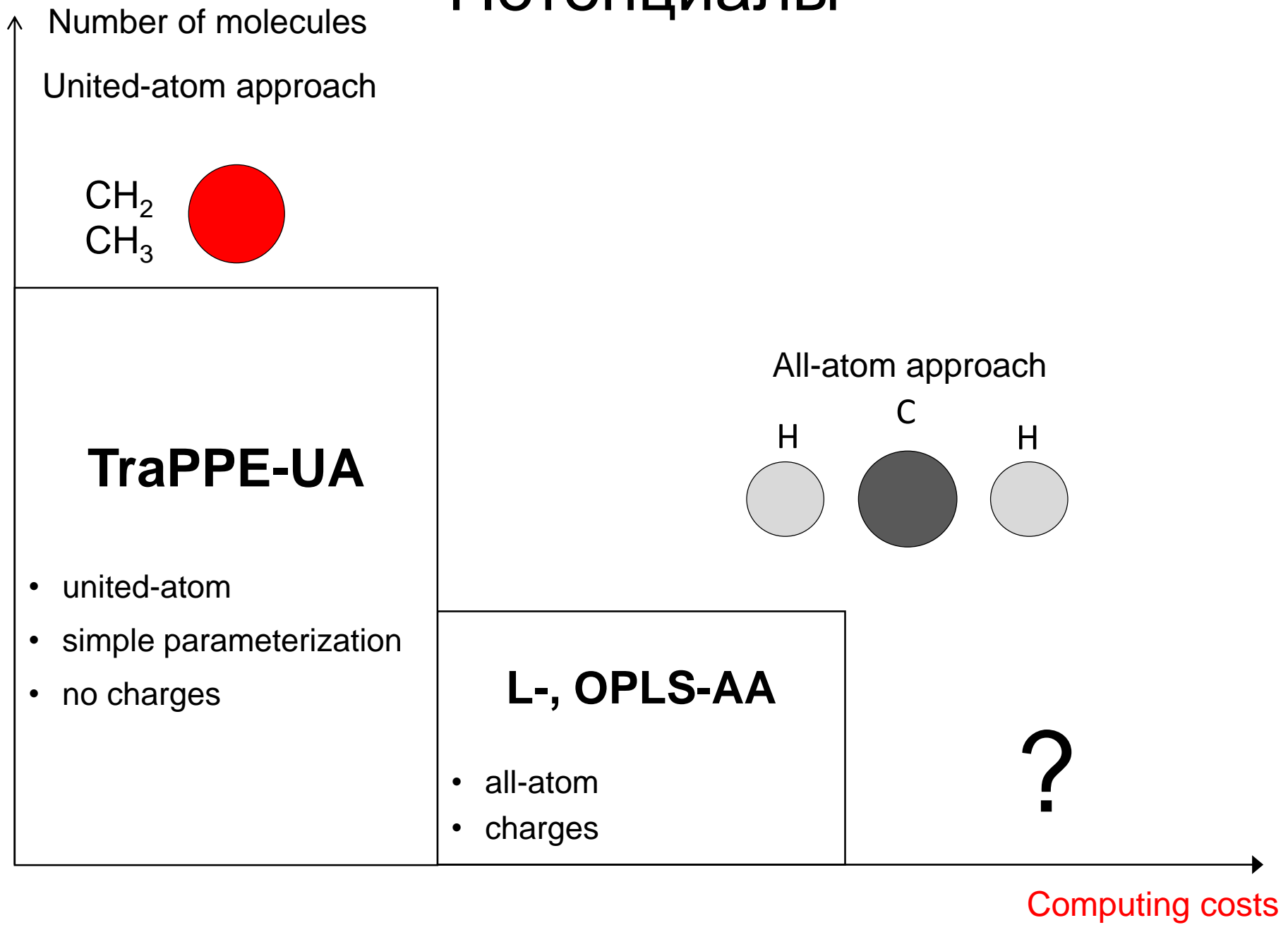
**8485 citations**

## L-OPLS-AA (2012)

*Siu, S. W. I., Pluhackova, K., Bockmann, R. A.* Optimization of the OPLS-AA Force Field for Long Hydrocarbons // J. Chem. Theory Comput. 2012. V. 8. P. 1459.

**168 citations**

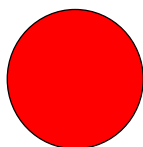
# Потенциалы



# Потенциалы

Number of molecules  
United-atom approach

CH<sub>2</sub>  
CH<sub>3</sub>



## TraPPE-UA

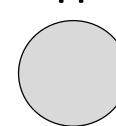
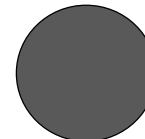
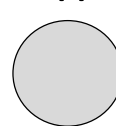
- united-atom
- simple parameterization
- no charges

All-atom approach

H

C

H



## L-, OPLS-AA

- all-atom
- charges

## COMPASS

- difficult parameterization

Computing costs

# Модель класса II

## COMPASS (1998)

Sun, H.

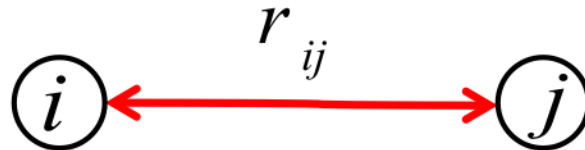
COMPASS: An ab Initio Force-Field Optimized for Condensed-Phase Applications s  
Overview with Details on Alkane and Benzene Compounds.

*J. Phys. Chem.*, 5647(98), 7338–7364

**3285 citations**

$$E = E_{bond} + E_{angle} + E_{dihedral} + E_{LJ} + E_{Coul}$$

$$E = K_2(r - r_0)^2 + K_3(r - r_0)^3 + K_4(r - r_0)^4$$



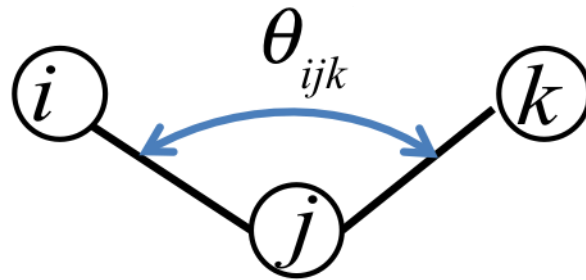
# Модель класса II

$$E = E_a + E_{bb} + E_{ba}$$

$$E_a = K_2(\theta - \theta_0)^2 + K_3(\theta - \theta_0)^3 + K_4(\theta - \theta_0)^4$$

$$E_{bb} = M(r_{ij} - r_1)(r_{jk} - r_2)$$

$$E_{ba} = N_1(r_{ij} - r_1)(\theta - \theta_0) + N_2(r_{jk} - r_2)(\theta - \theta_0)$$





# Модель класса II

$$E = E_d + E_{mbt} + E_{ebt} + E_{at} + E_{aat} + E_{bb13}$$

$$E_d = \sum_{n=1}^3 K_n [1 - \cos(n\phi - \phi_n)]$$

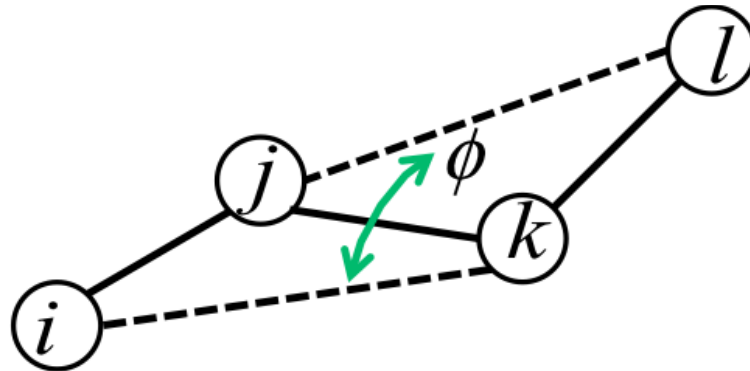
$$E_{mbt} = (r_{jk} - r_2) [A_1 \cos(\phi) + A_2 \cos(2\phi) + A_3 \cos(3\phi)]$$

$$E_{ebt} = (r_{ij} - r_1) [B_1 \cos(\phi) + B_2 \cos(2\phi) + B_3 \cos(3\phi)] + (r_{kl} - r_3) [C_1 \cos(\phi) + C_2 \cos(2\phi) + C_3 \cos(3\phi)]$$

$$E_{at} = (\theta_{ijk} - \theta_1) [D_1 \cos(\phi) + D_2 \cos(2\phi) + D_3 \cos(3\phi)] + (\theta_{jkl} - \theta_2) [E_1 \cos(\phi) + E_2 \cos(2\phi) + E_3 \cos(3\phi)]$$

$$E_{aat} = M(\theta_{ijk} - \theta_1)(\theta_{jkl} - \theta_2) \cos(\phi)$$

$$E_{bb13} = N(r_{ij} - r_1)(r_{kl} - r_3)$$



+ Lennard-Jones **6-9** + Coulomb

```

set type 1 charge -0.159000
set type 2 charge 0.000000
set type 3 charge -0.053000
set type 4 charge 0.053000
set type 5 charge -0.106000

pair_style lj/class2/coul/long 12.0
pair_modify tail yes mix sixthpower
kspace_style pppm 0.00001
special_bonds lj/coul 0.0 0.0 1.0 dihedral yes
pair_coeff 1 1 0.0620000000 3.8540000000 # c4
pair_coeff 2 2 0.0200000000 3.8540000000 # c44
pair_coeff 3 3 0.0400000000 3.8540000000 # c43
pair_coeff 4 4 0.0230000000 2.8780000000 # h1
pair_coeff 5 5 0.0620000000 3.8540000000 # c4

bond_style class2
bond_coeff 1 1 1.5300 299.6700 -501.7700 679.8100 # c4-c44
bond_coeff 2 1 1.1010 345.0000 -691.8900 844.6000 # c4-h1
bond_coeff 3 1 1.5300 299.6700 -501.7700 679.8100 # c4-c43
bond_coeff 4 1 1.1010 345.0000 -691.8900 844.6000 # c43-h1

angle_style class2
angle_coeff 1 110.7700 41.4530 -10.6040 5.1290 # c44-c4-h1
angle_coeff 2 107.6600 39.6410 -12.9210 -2.4318 # h1-c4-h1
angle_coeff 3 112.6700 39.5160 -7.4430 -9.5583 # c4-c44-c4
angle_coeff 4 112.6700 39.5160 -7.4430 -9.5583 # c44-c4-c43
angle_coeff 5 110.7700 41.4530 -10.6040 5.1290 # c43-c4-h1
angle_coeff 6 112.6700 39.5160 -7.4430 -9.5583 # c4-c43-c4
angle_coeff 7 110.7700 41.4530 -10.6040 5.1290 # c4-c43-h1

dihedral_style class2
dihedral_coeff 1 0.0000 0.0000 0.0316 0.0000 -0.1681 0.0000# h1-c4-c44-c4
dihedral_coeff 2 0.0000 0.0000 0.0514 0.0000 -0.1430 0.0000# c43-c4-c44-c4
dihedral_coeff 3 0.0000 0.0000 0.0514 0.0000 -0.1430 0.0000# c44-c4-c43-c4
dihedral_coeff 4 0.0000 0.0000 0.0316 0.0000 -0.1681 0.0000# c44-c4-c43-h1
dihedral_coeff 5 0.0000 0.0000 0.0316 0.0000 -0.1681 0.0000# h1-c4-c43-c4
dihedral_coeff 6 -0.1432 0.0000 0.0617 0.0000 -0.1530 0.0000# h1-c4-c43-h1

improper_style class2
improper_coeff 1 0.0000 0.0000
improper_coeff 2 0.0000 0.0000
improper_coeff 3 0.0000 0.0000
improper_coeff 4 0.0000 0.0000
improper_coeff 5 0.0000 0.0000
improper_coeff 6 0.0000 0.0000
improper_coeff 7 0.0000 0.0000

angle_coeff 1 bb 3.3872 1.5300 1.1010
angle_coeff 2 bb 5.3316 1.1010 1.1010
angle_coeff 3 bb 0.0000 1.5300 1.5300
angle_coeff 4 bb 0.0000 1.5300 1.5300
angle_coeff 5 bb 3.3872 1.5300 1.1010
angle_coeff 6 bb 0.0000 1.5300 1.5300
angle_coeff 7 bb 3.3872 1.5300 1.1010

angle_coeff 1 ba 20.7540 11.4210 1.5300 1.1010
angle_coeff 2 ba 18.1030 18.1030 1.1010 1.1010
angle_coeff 3 ba 8.0160 8.0160 1.5300 1.5300
angle_coeff 4 ba 8.0160 8.0160 1.5300 1.5300
angle_coeff 5 ba 20.7540 11.4210 1.5300 1.1010

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angle_coeff 4	ba	8.0160	8.0160	1.5300	1.5300		
angle_coeff 5	ba	20.7540	11.4210	1.5300	1.1010		
angle_coeff 6	ba	8.0160	8.0160	1.5300	1.5300		
angle_coeff 7	ba	20.7540	11.4210	1.5300	1.1010		
improper_coeff 1	aa	0.2738	-0.4825	0.2738	110.7700	107.6600	110.7700
improper_coeff 2	aa	-0.3157	-0.3157	-0.3157	107.6600	107.6600	107.6600
improper_coeff 3	aa	-0.1729	-0.1729	-0.1729	112.6700	112.6700	112.6700
improper_coeff 4	aa	-1.3199	-1.3199	0.1184	112.6700	110.7700	110.7700
improper_coeff 5	aa	0.2738	-0.4825	0.2738	110.7700	107.6600	110.7700
improper_coeff 6	aa	-0.1729	-0.1729	-0.1729	112.6700	112.6700	112.6700
improper_coeff 7	aa	-1.3199	-1.3199	0.1184	112.6700	110.7700	110.7700
dihedral_coeff 1	aat	-16.1640	110.7700	112.6700			
dihedral_coeff 2	aat	-22.0450	112.6700	112.6700			
dihedral_coeff 3	aat	-22.0450	112.6700	112.6700			
dihedral_coeff 4	aat	-16.1640	112.6700	110.7700			
dihedral_coeff 5	aat	-16.1640	110.7700	112.6700			
dihedral_coeff 6	aat	-12.5640	110.7700	110.7700			
dihedral_coeff 1	ebt	0.0814	0.0591	0.2219	0.2486	0.2422	-0.0900
dihedral_coeff 2	ebt	-0.0732	0.0000	0.0000	-0.0732	0.0000	0.0000
dihedral_coeff 3	ebt	-0.0732	0.0000	0.0000	-0.0732	0.0000	0.0000
dihedral_coeff 4	ebt	0.2486	0.2422	-0.0925	0.0814	0.0591	0.2219
dihedral_coeff 5	ebt	0.0814	0.0591	0.2219	0.2486	0.2422	-0.0900
dihedral_coeff 6	ebt	0.2130	0.3120	0.0777	0.2130	0.3120	0.0777
dihedral_coeff 1	mbt	-14.8790	-3.6581	-0.3138	1.5300		
dihedral_coeff 2	mbt	-17.7870	-7.1877	0.0000	1.5300		
dihedral_coeff 3	mbt	-17.7870	-7.1877	0.0000	1.5300		
dihedral_coeff 4	mbt	-14.8790	-3.6581	-0.3138	1.5300		
dihedral_coeff 5	mbt	-14.8790	-3.6581	-0.3138	1.5300		
dihedral_coeff 6	mbt	-14.2610	-0.5322	-0.4864	1.5300		
dihedral_coeff 1	bb13	0.0000	1.1010	1.5300			
dihedral_coeff 2	bb13	0.0000	1.5300	1.5300			
dihedral_coeff 3	bb13	0.0000	1.5300	1.5300			
dihedral_coeff 4	bb13	0.0000	1.5300	1.1010			
dihedral_coeff 5	bb13	0.0000	1.1010	1.5300			
dihedral_coeff 6	bb13	0.0000	1.1010	1.1010			
dihedral_coeff 1	at	0.3113	0.4516	-0.1988	-0.2454	0.0000	-0.1100
dihedral_coeff 2	at	0.3886	-0.3139	0.1389	0.3886	-0.3139	0.1389
dihedral_coeff 3	at	0.3886	-0.3139	0.1389	0.3886	-0.3139	0.1389
dihedral_coeff 4	at	-0.2454	0.0000	-0.1136	0.3113	0.4516	-0.1988
dihedral_coeff 5	at	0.3113	0.4516	-0.1988	-0.2454	0.0000	-0.1100
dihedral_coeff 6	at	-0.8085	0.5569	-0.2466	-0.8085	0.5569	-0.2466

United-atom approach  
 TraPPE-UA  
 • united-atom  
 • simple parameterization  
 • no charges

All-atom approach  
 L-OPLS-AA  
 COMPASS  
 • all-atom  
 • charges

# Система

Потенциал объединенного атома

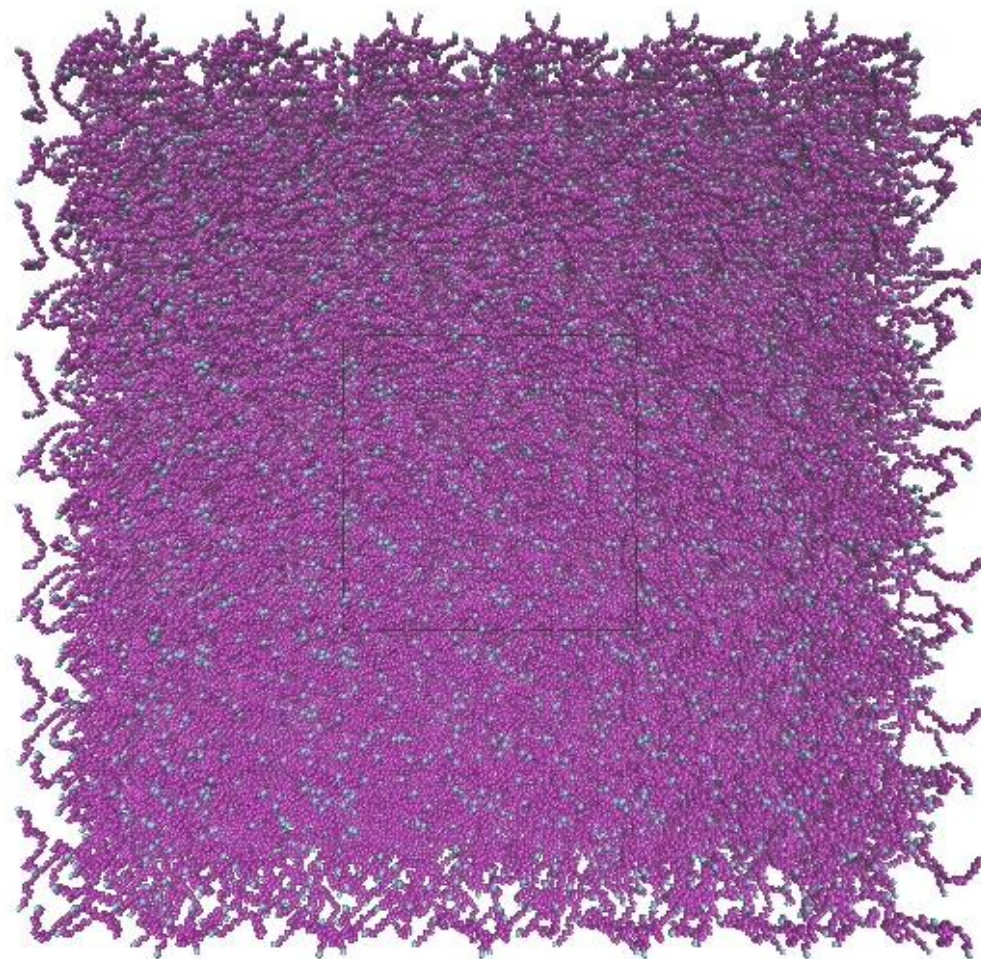


● Центры масс



$$\langle \Delta r^2 \rangle$$

$$\langle v(0)v(t) \rangle$$



P ~ 1 atm    T = 360 ÷ 500 K

L = 19.2 nm    N = 800k

\*LAMMPS



# Среднеквадратичные смещения

Ballistic regime

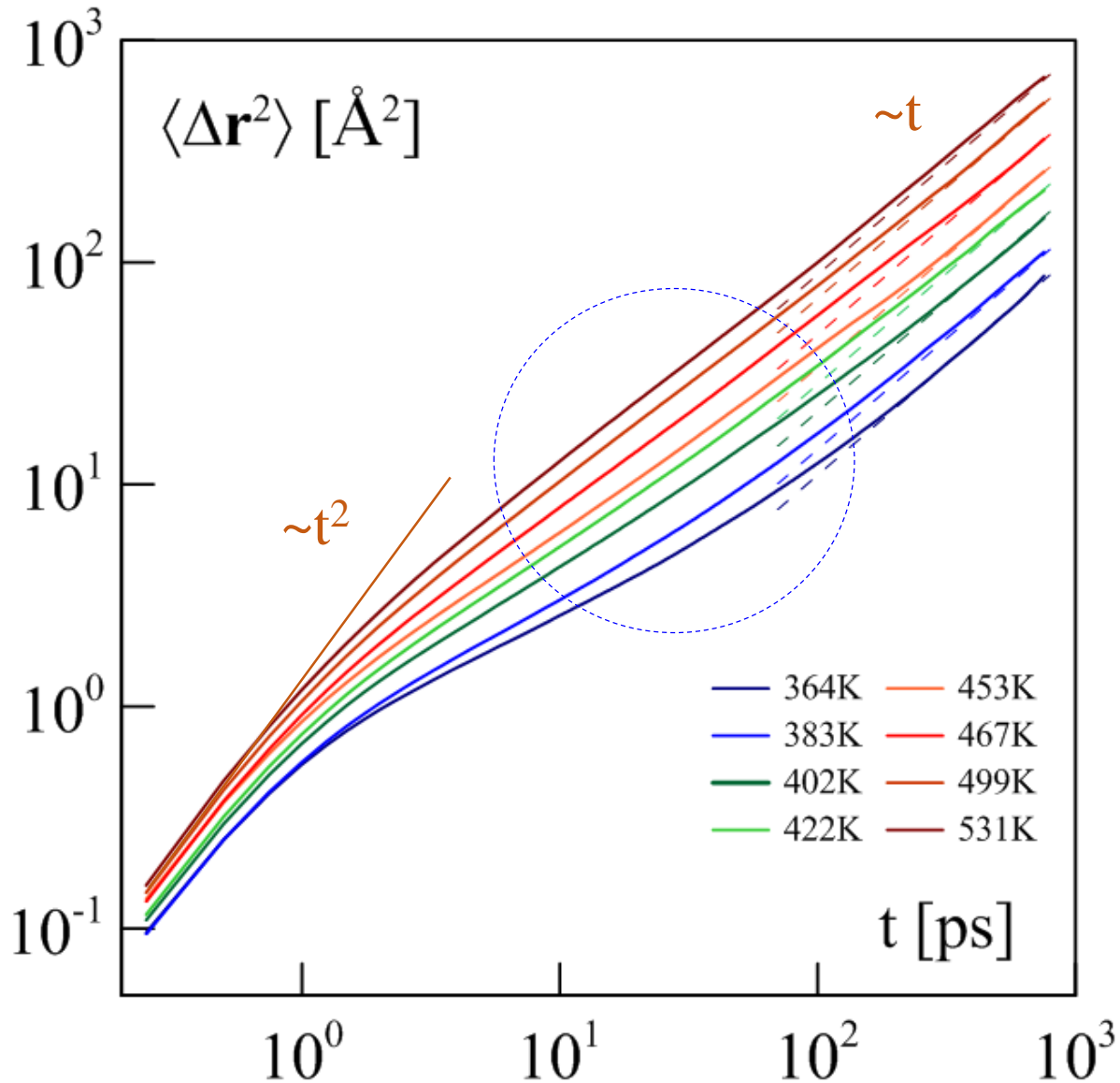
—  $\langle \Delta r^2 \rangle = v^2 t^2$

Subdiffusive regime

- - -  $\langle \Delta r^2 \rangle \sim t^\alpha, \alpha < 1$

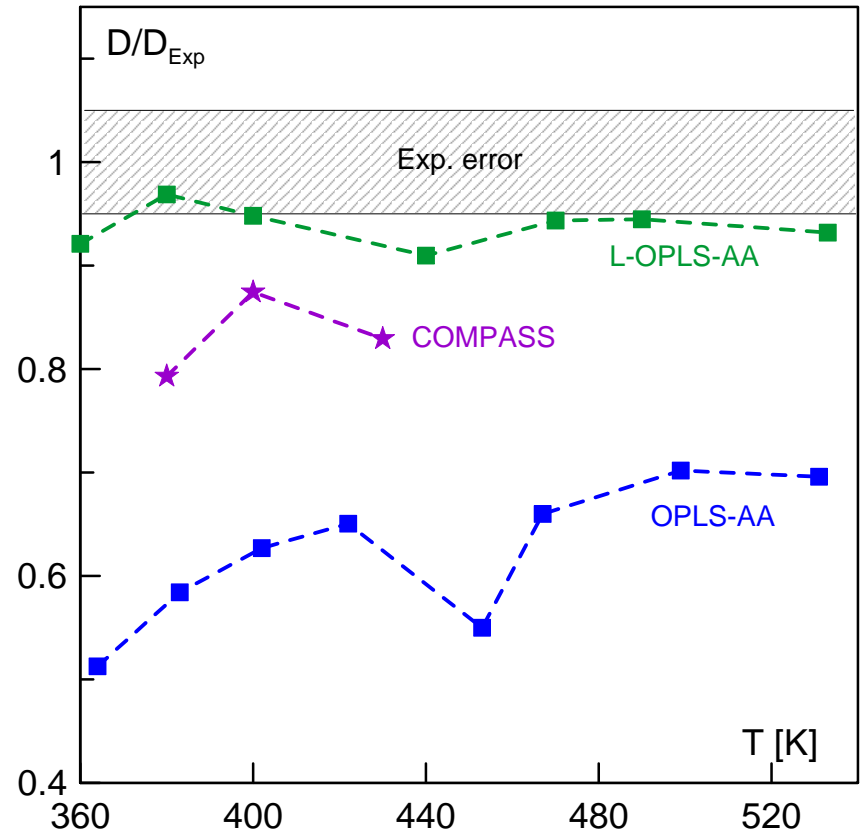
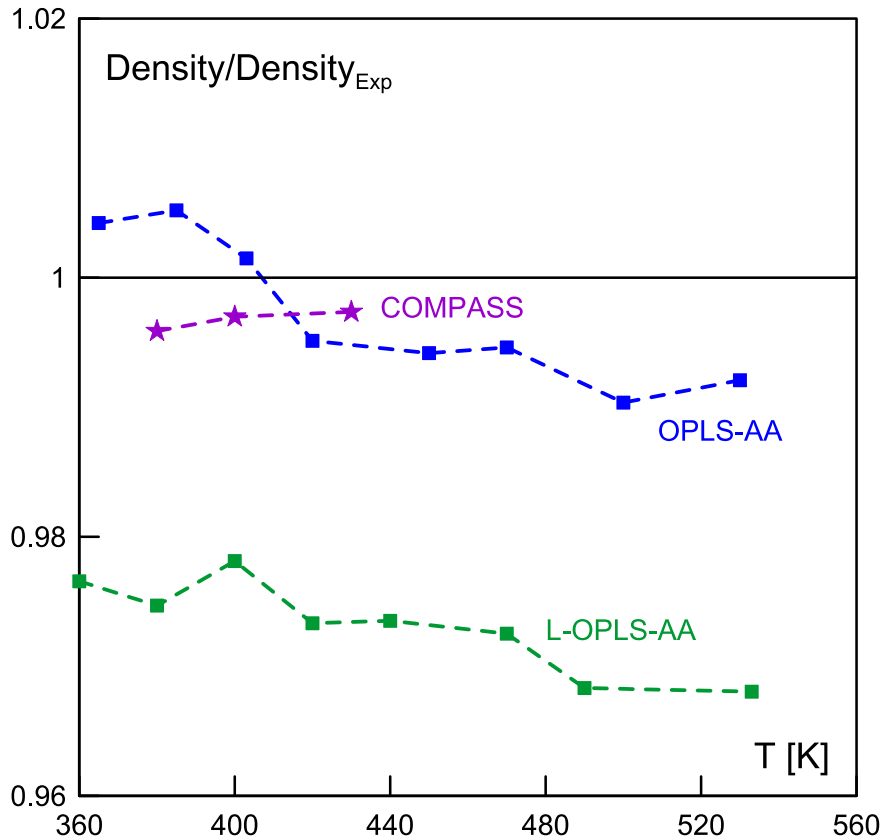
Diffusive regime

- - -  $\langle \Delta r^2 \rangle = 6Dt$



# Предсказательная способность

Kondratyuk N., Norman G. and Stegailov V. *J. Chem. Phys.*, 2016, **145**, 204504.



**COMPASS (модель класса 2):** **корректные** плотность и диффузия

\* В.М. Татаевский. Физико-химические свойства индивидуальных углеводородов. Москва: ГОСТОПТЕХИЗДАТ, 1960. 413 с.

\*\*T. Vardag, N. Karger, H.D. Lüdemann // Ber. Bunsenges. Phys. Chem. 1991. V. 95. N. 8. P. 859.

# План доклада

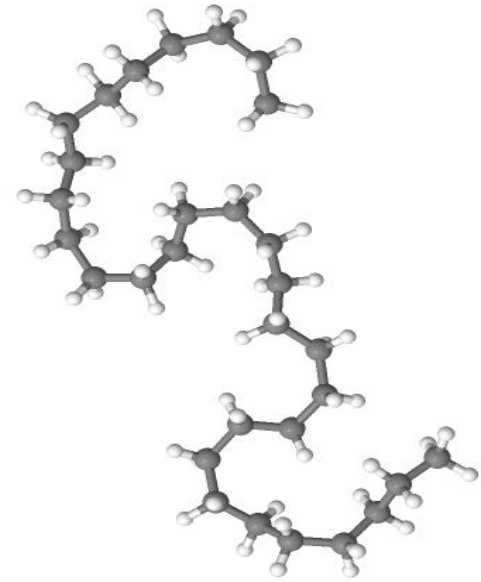


## 1. Самодиффузия в жидком *n*-триаконтане

Потенциалы взаимодействия (+ COMPASS)

Сходимость Г-К

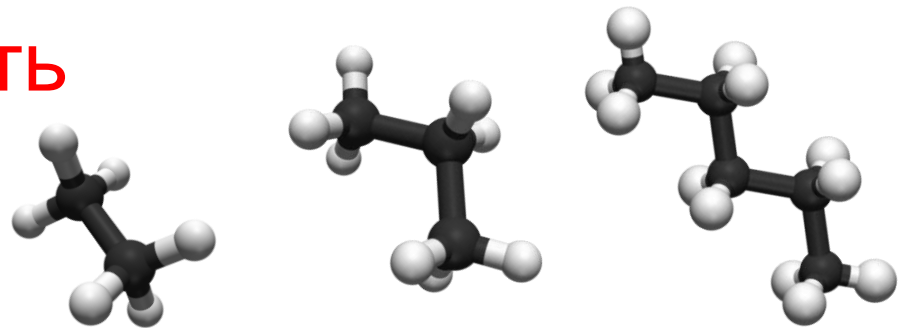
Предсказательная способность *Молекула *n*-триаконтана*



## 2. Сдвиговая вязкость

Сходимость Г-К

Неравновесная МД + эксперимент



# Мотивация

## 1. РФФ 17-79-20391 (В.В. Писарев)

Атомистическое и многомасштабное моделирование фильтрации флюидов в пористой среде

## 2. “Старые счеты”: проблемы со сходимостью Г-К

Table 1. Physical properties for the different models at 353 K and 0.775 g/cm<sup>3</sup>.

Model	TraPPE	DREIDING	OPLS	Exp
Pressure, bar	1	4200	1	1
Shear viscosity, mPas	0.96	4.33	—	4.87 [22]

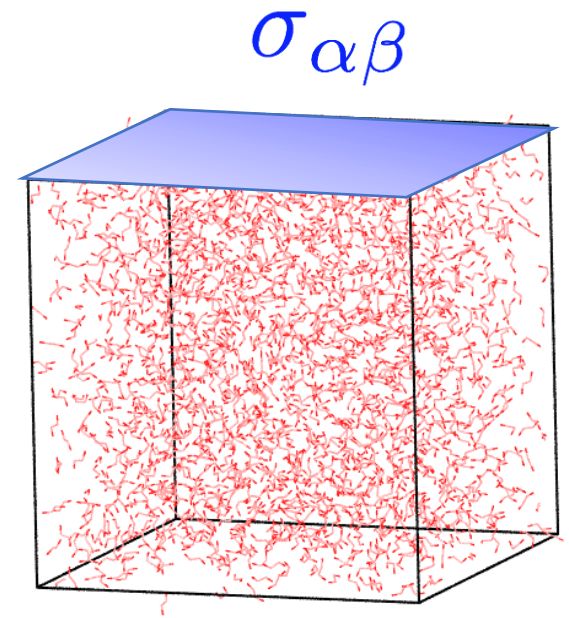
Kondratyuk, N. D., Lankin, A. V, Norman, G. E. and Stegailov, V. V. (2015). J. Phys. Conf. Ser. 653. 12107.

# Метод Грина-Кубо для вязкости

$$\eta_{\alpha\beta} = \frac{V}{k_B T} \int_0^{\infty} C_{\sigma}(t) dt$$

$$C_{\sigma}(t) = \langle \sigma_{\alpha\beta}(0) \sigma_{\alpha\beta}(t) \rangle$$

$$\sigma_{\alpha\beta} V = \sum_{i=1}^N m_i v_{i\alpha} v_{i\beta} + \sum_{i=1}^N r_{i\alpha} f_{i\beta}$$





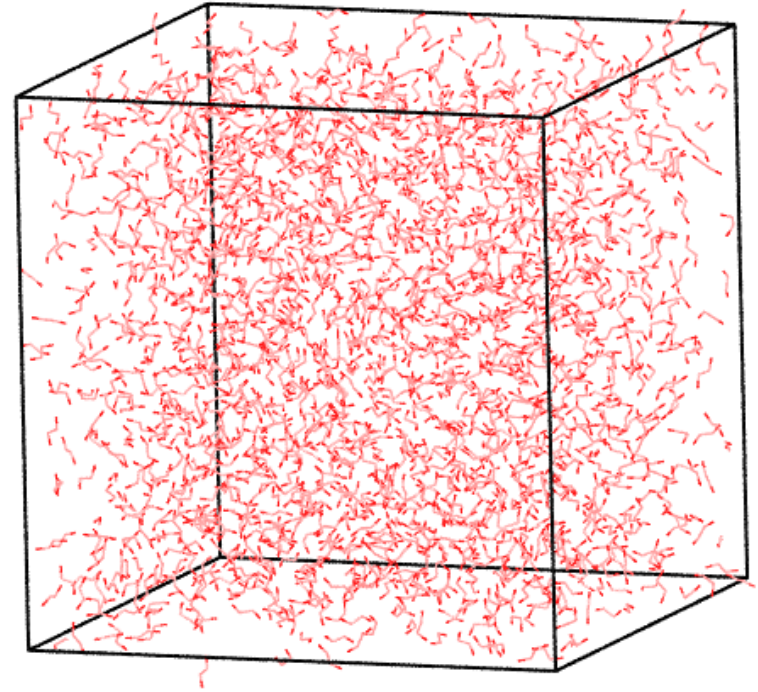
# Рассматриваемые системы

Этан, пропан, бутан, пентан

Потенциал OPLS-AA (полноатомный)

NVE, интегратор rRESPA  
(1 фс – шаг для электростатики и LJ)

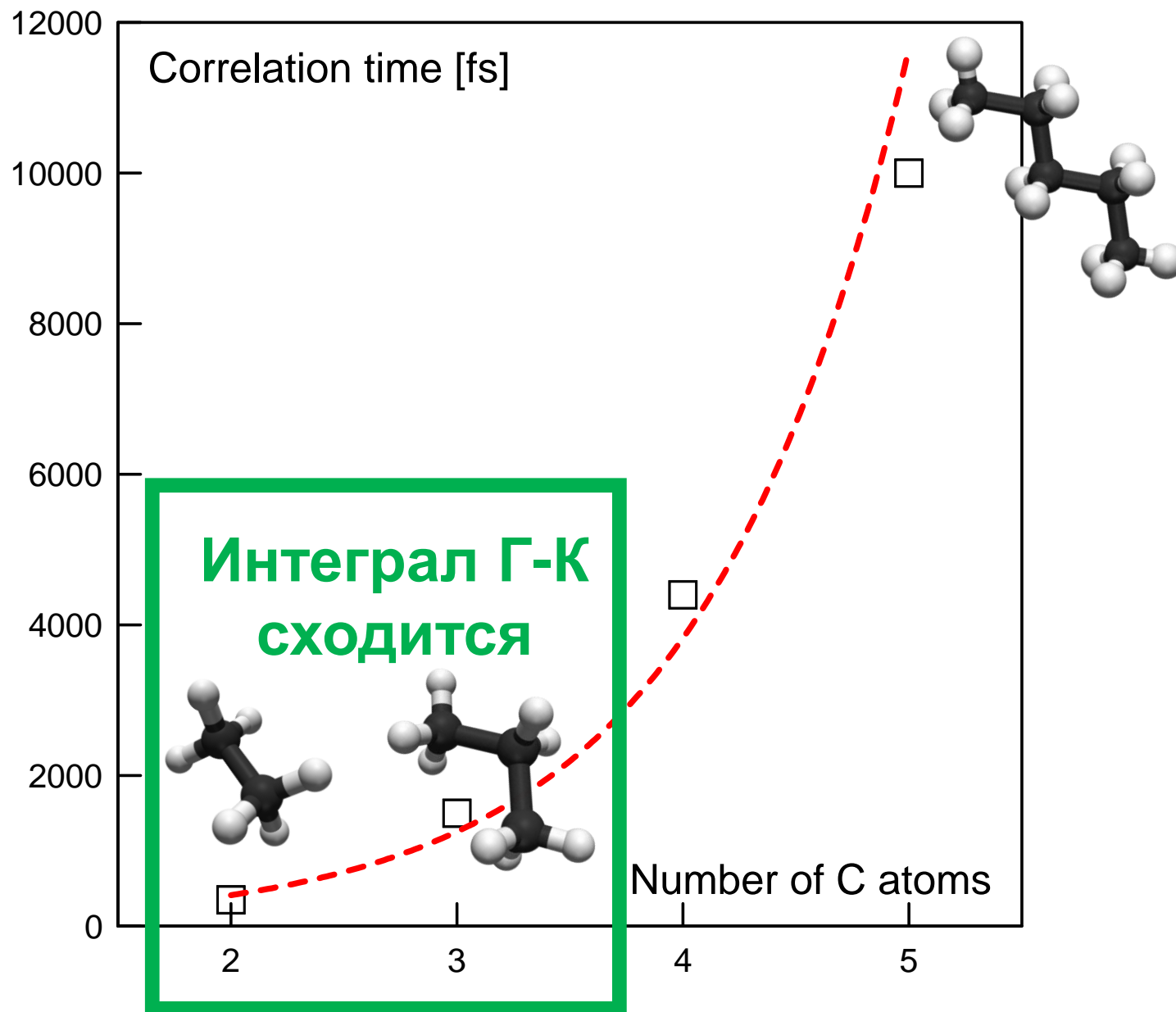
Длины траекторий - 6 нс



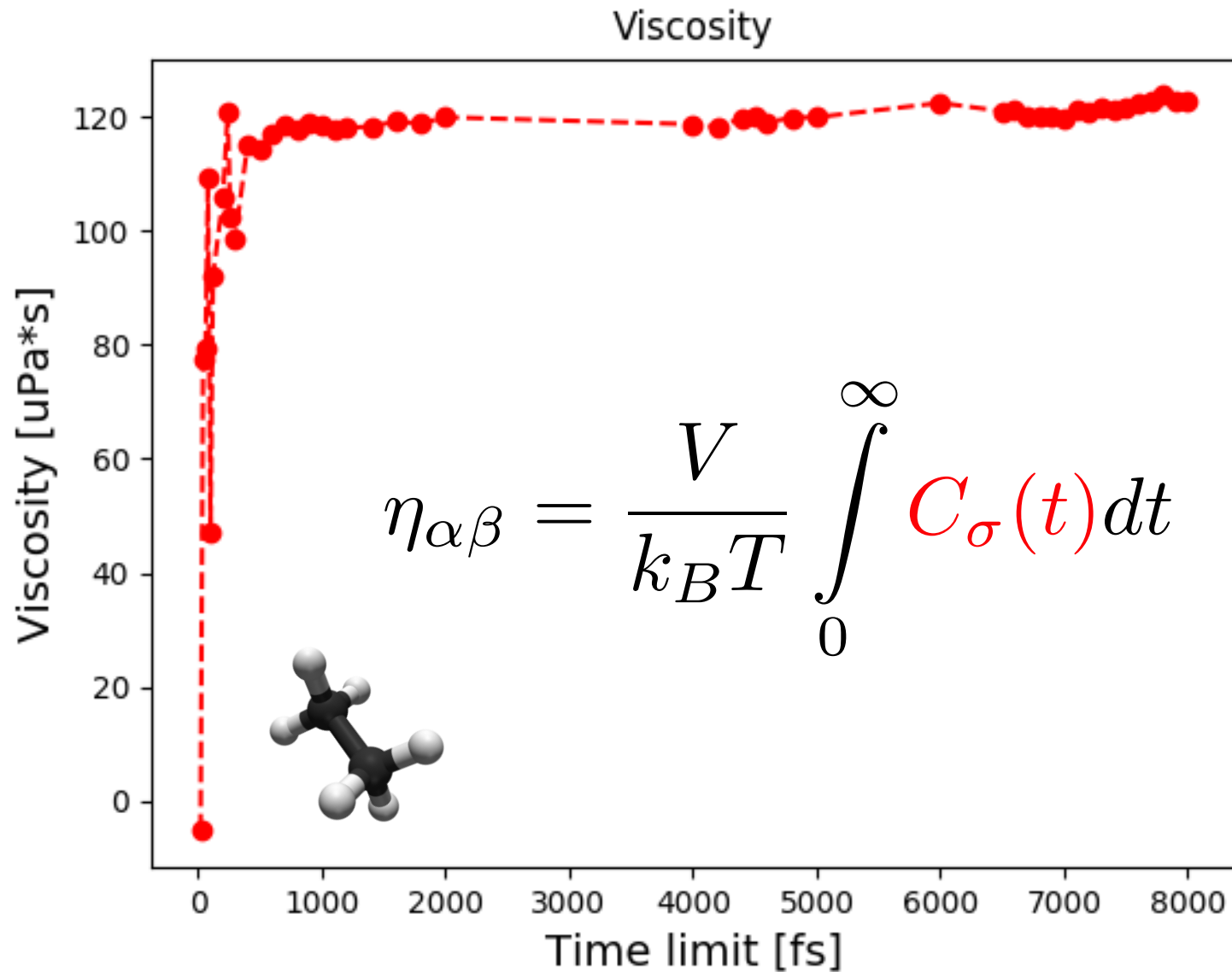
$$T = 330 \text{ K} \quad \rho = 0.5 \text{ g/cm}^3$$

$$L \sim 6 \text{ nm} \quad N = 80\text{k}$$

# Время затухания АКФ



# Сходимость интеграла Грина-Кубо

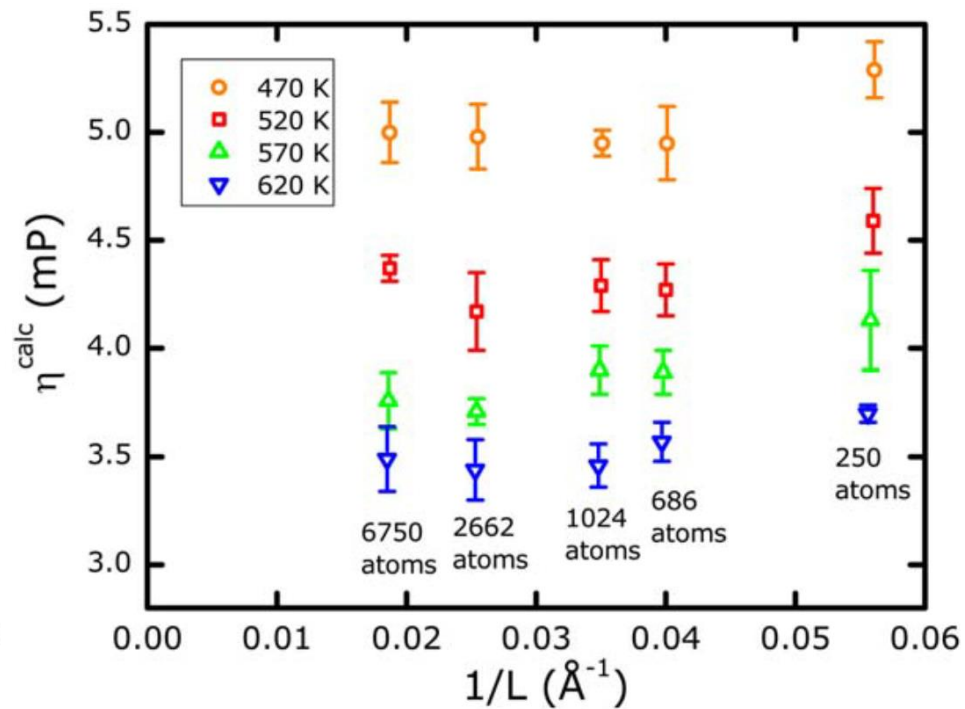
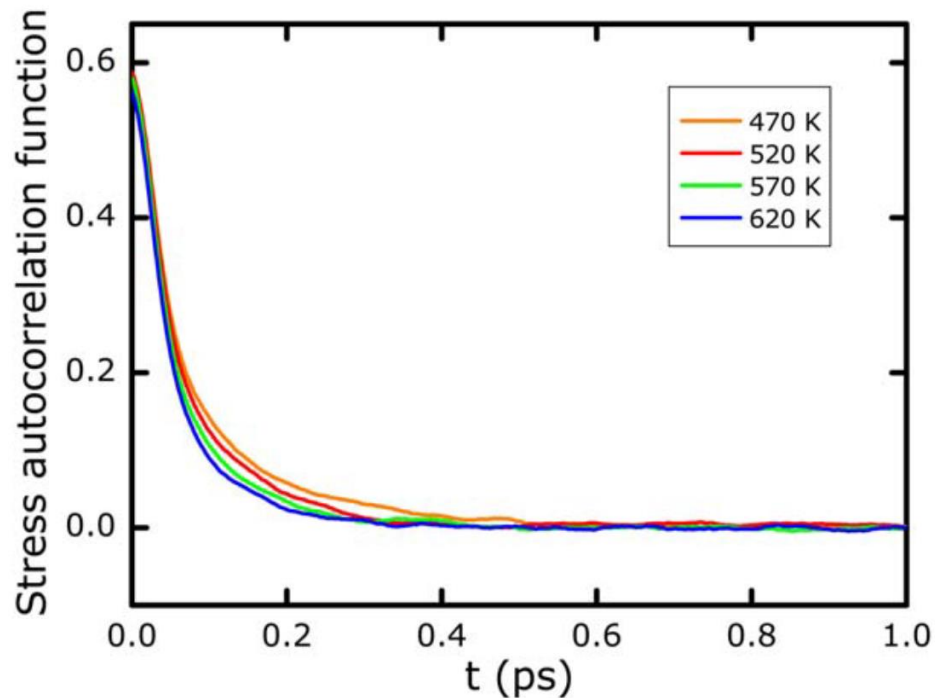


# Сходимость интеграла Грина-Кубо

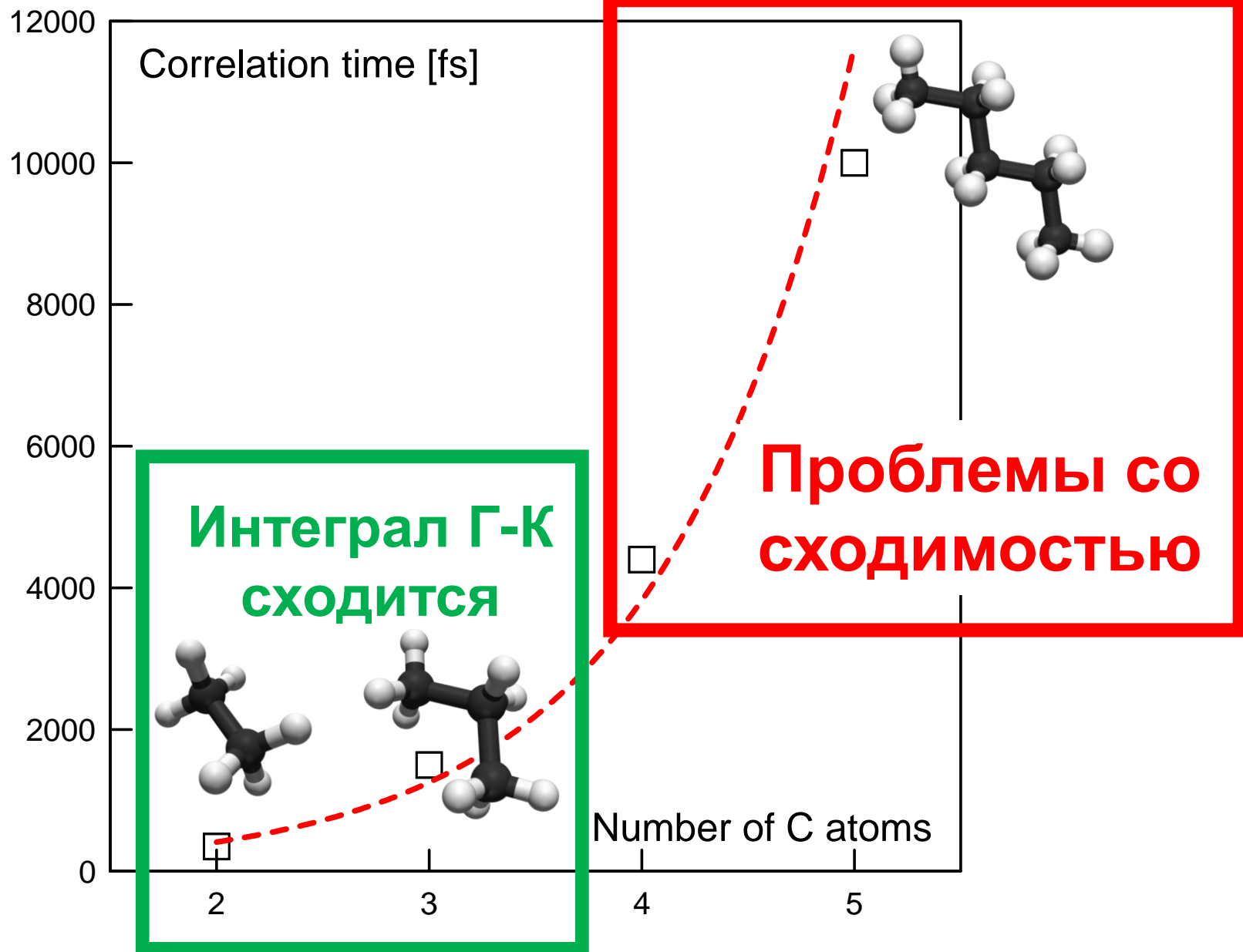
Chen, M. , Vella, J. R., Panagiotopoulos, A. Z., Debenedetti, P. G., Stillinger, F. H. and Carter, E. A.

Liquid Li structure and dynamics: A comparison between OFDFT and second nearest-neighbor embedded-atom method.

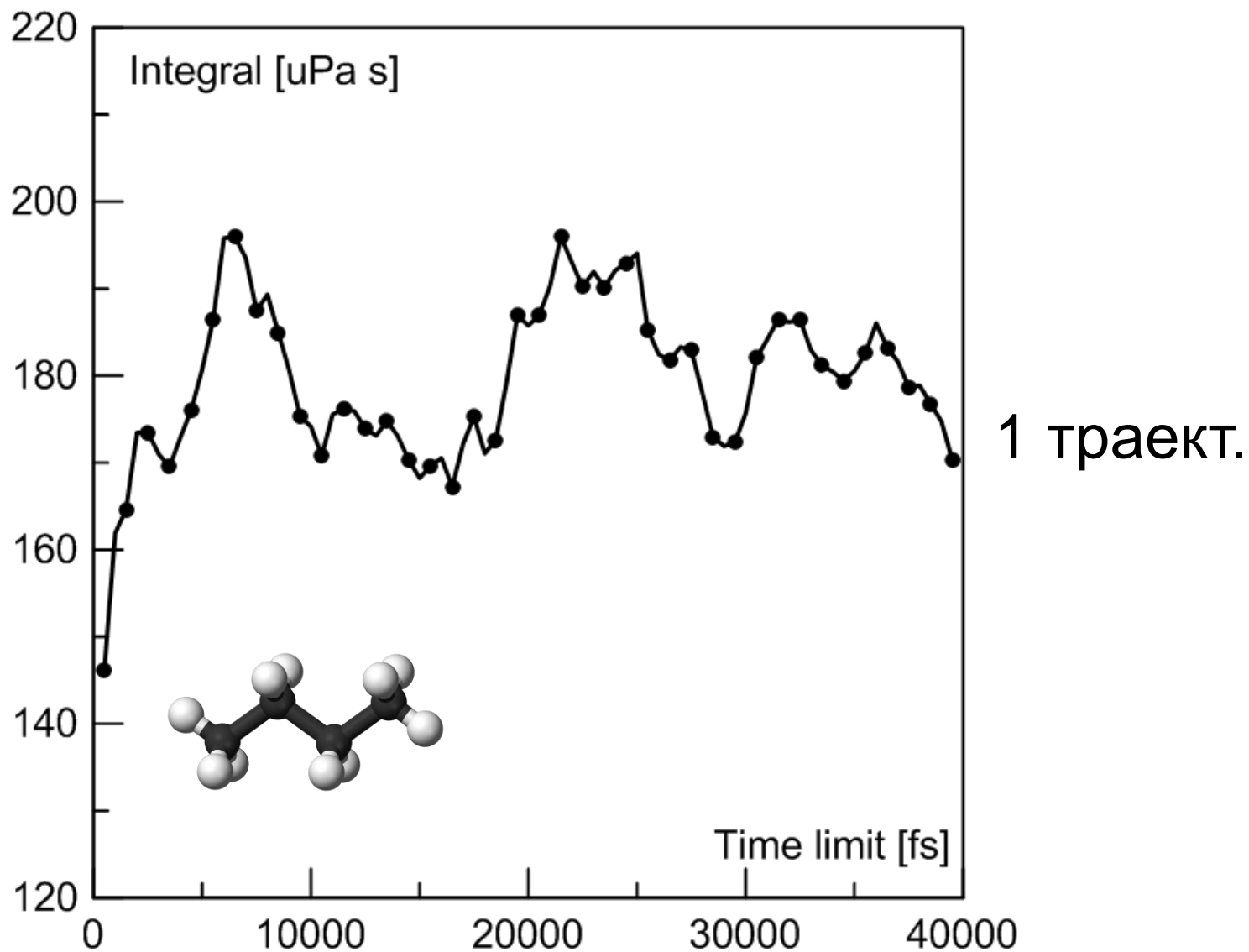
AIChE J., (2015), 61: 2841-2853



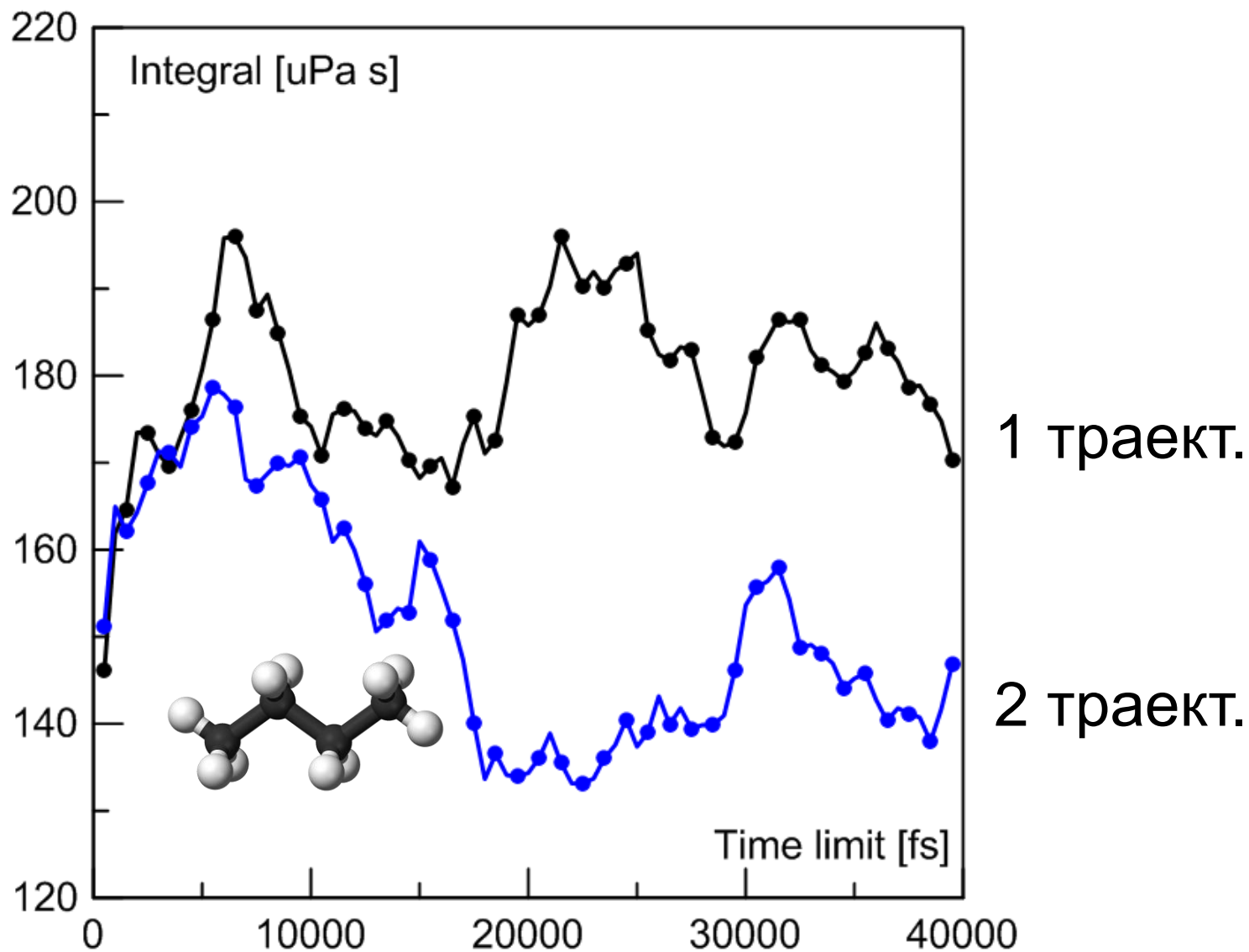
# Время затухания АКФ



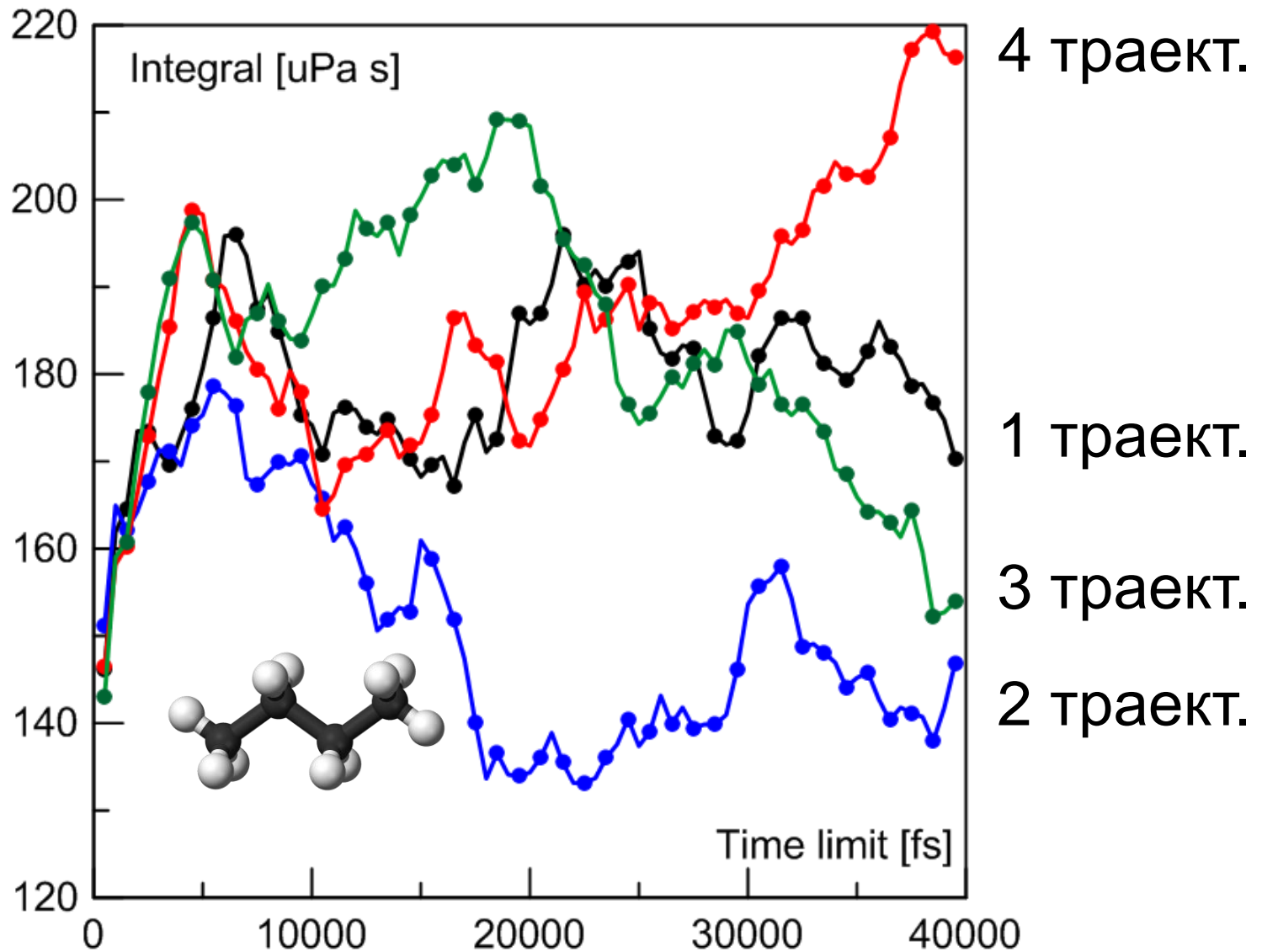
# Сходимость интеграла Грина-Кубо



# Сходимость интеграла Грина-Кубо



# Сходимость интеграла Грина-Кубо



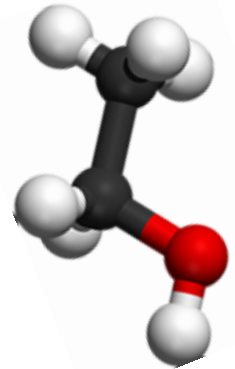
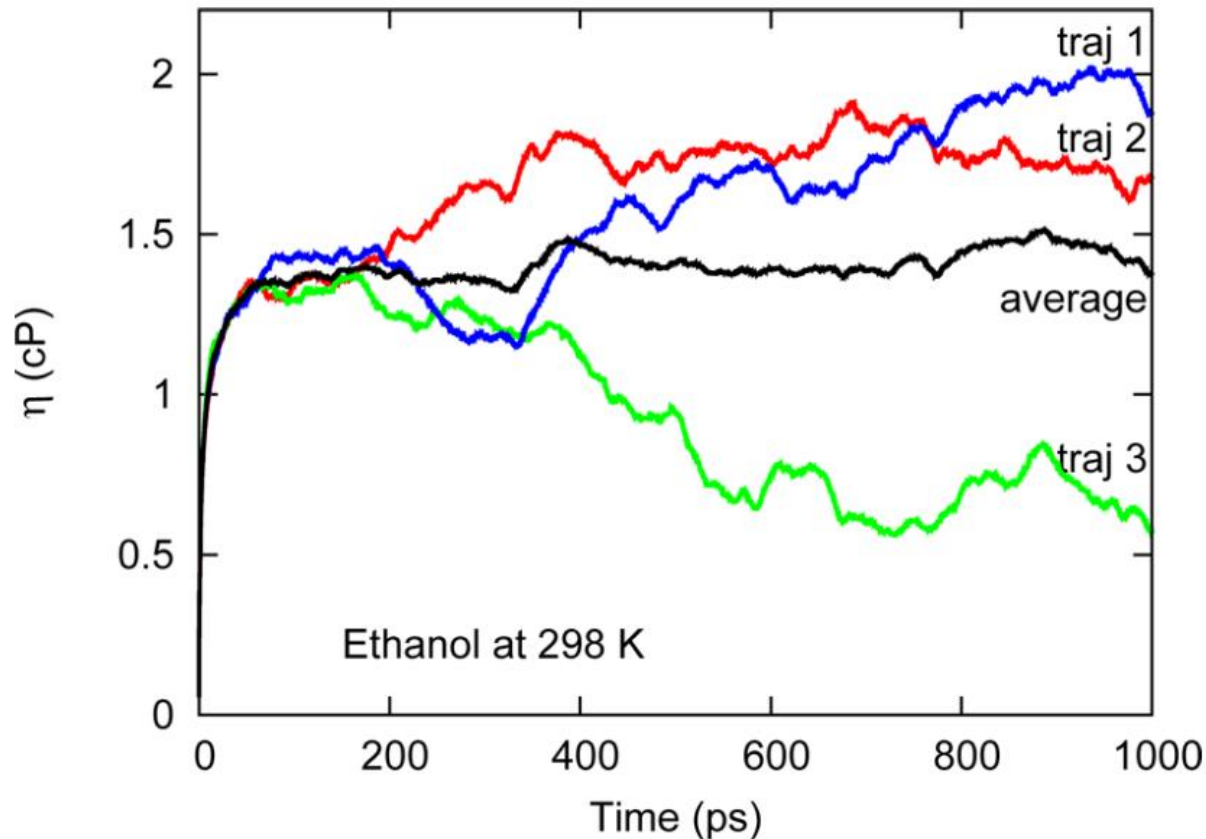


# Сходимость интеграла Грина-Кубо

Zhang, Y., Otani, A. and Maginn, E. J.

Reliable Viscosity Calculation from Equilibrium Molecular Dynamics Simulations: A Time Decomposition Method.

*J. Chem. Theory Comput.*, (2015), 11(8), 3537–3546.



# Time decomposition method

- (1) Generate  $N$  independent NVT trajectories at a given temperature;
- (2) For each trajectory, calculate the shear viscosity based on the Green–Kubo relation (eq 1);
- (3) Calculate the average of the running integrals over  $N$  trajectories  $\langle \eta(t) \rangle$  and the standard deviation, which is a function of time:

$$\sigma(t) = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (\eta(t)_i - \langle \eta(t) \rangle)^2}$$

# Time decomposition method

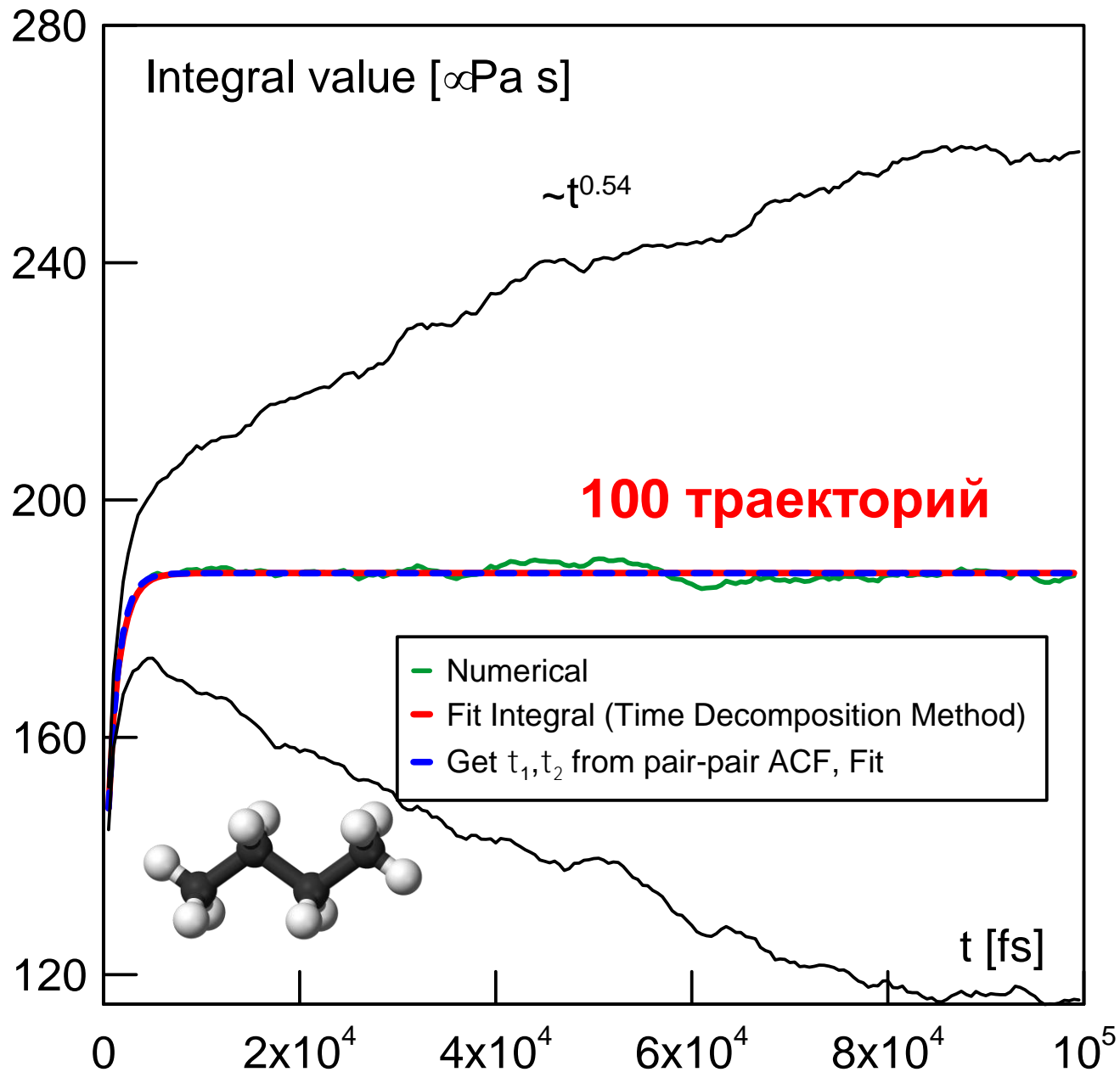
- (4) Fit the standard deviation to a power law function

$$\sigma(t) = At^b \quad (4)$$

- (5) Fit the averaged running integral by the double-exponential function (eq 2) for the time period up to  $t_{\text{cut}}$  with the weight  $1/t^b$ , where  $b$  is the fitting result from step (4) and  $t_{\text{cut}}$  can be decided from the relation between  $\eta$  and  $\sigma(t)$ . We found that the time when  $\sigma(t)$  is about 40% of  $\langle\eta(t)\rangle$  is a good choice. Take the long time limit of the fitted double-exponential function as the calculated viscosity;

$$\eta(t) = A\alpha\tau_1(1 - e^{-t/\tau_1}) + A(1 - \alpha)\tau_2(1 - e^{-t/\tau_2})$$

# Time decomposition method



# План доклада

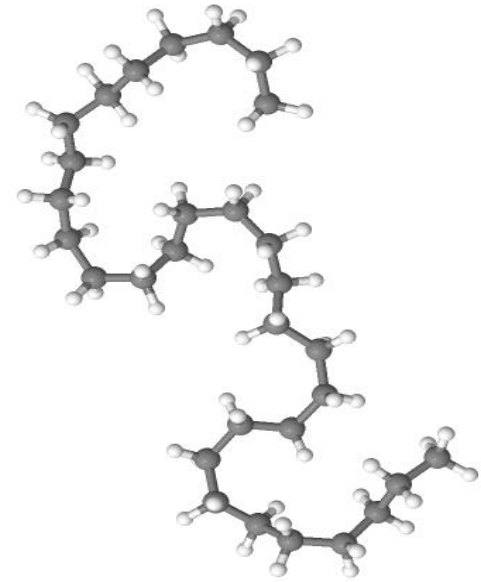


## 1. Самодиффузия в жидком *n*-триаконтане

Потенциалы взаимодействия (+ COMPASS)

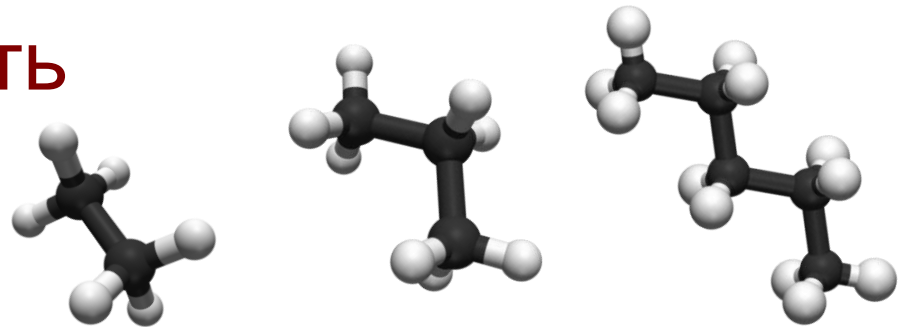
Сходимость Г-К

Предсказательная способность *Молекула *n*-триаконтана*



## 2. Сдвиговая вязкость

Сходимость Г-К



Неравновесная МД + эксперимент

# Метод неравновесной МД

Поток импульса

$$\eta = \frac{j_z(p_x)}{\partial v_x / \partial z}$$

Профиль скорости

**На практике:** обмен скоростями  $V_x$  каждые  $N$  шагов

1 слой 

1 слой 

$N/2 + 1$  слой 

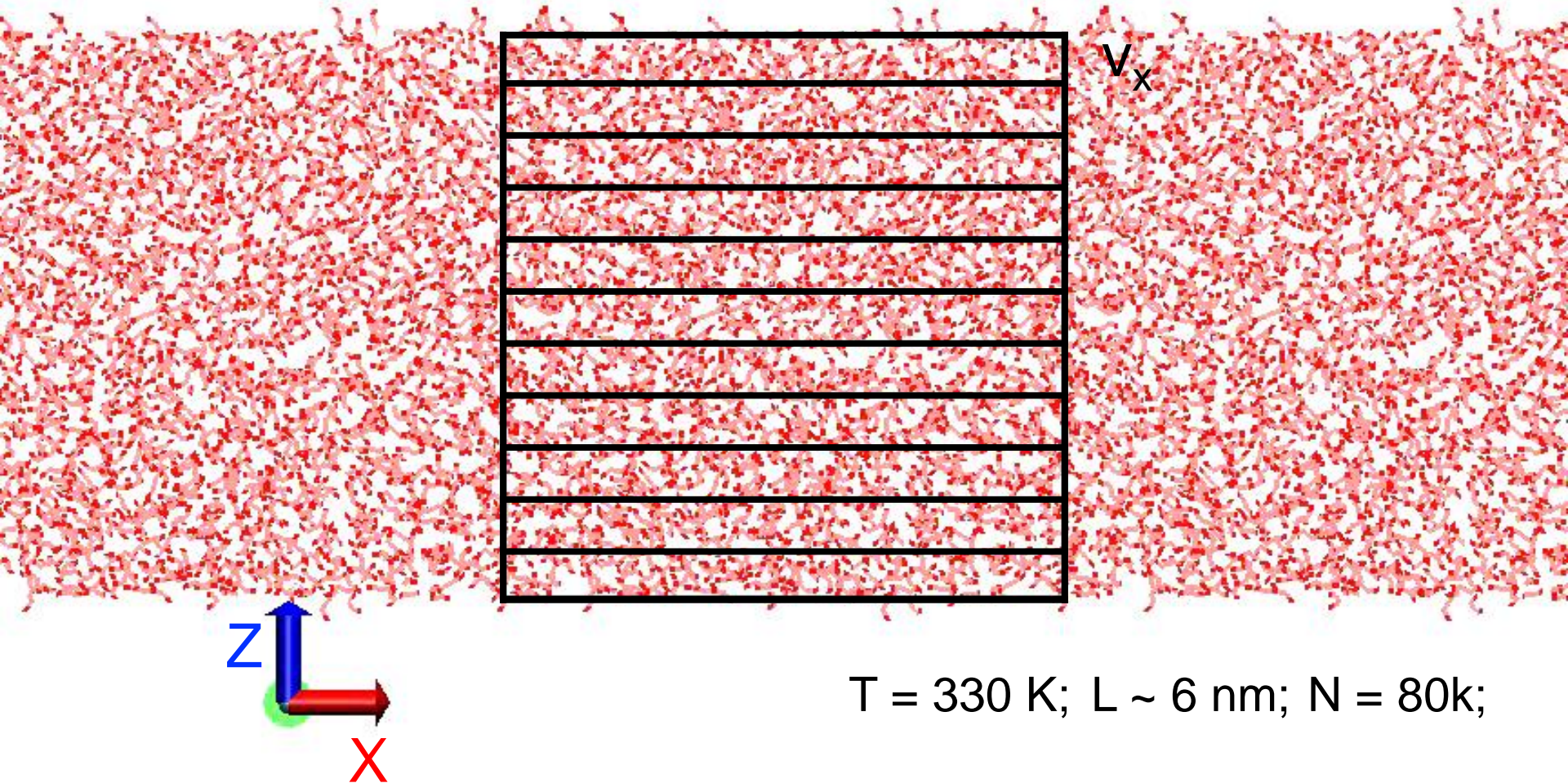
$N/2 + 1$  слой 

**(Muller-Plathe)** Muller-Plathe, Phys Rev E, 59, 4894-4898 (1999).



# Метод неравновесной МД

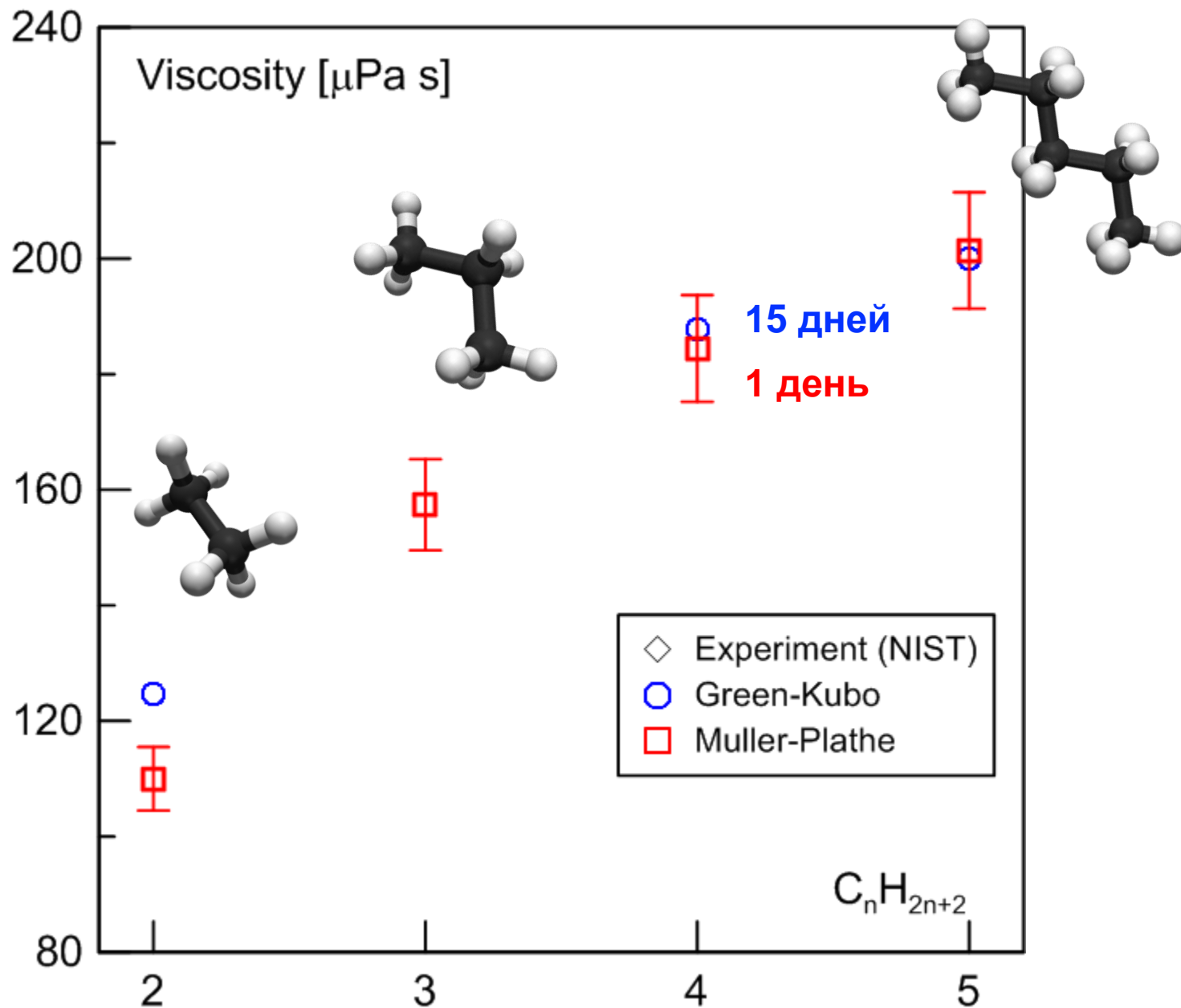
1 расчет = 1 день на узле Desmos (Intel Xeon E5-1650 v3 + NVIDIA GTX 1070)



$T = 330 \text{ K}; L \sim 6 \text{ nm}; N = 80\text{k};$

**(Muller-Plathe)** Muller-Plathe, Phys Rev E, 59, 4894-4898 (1999).

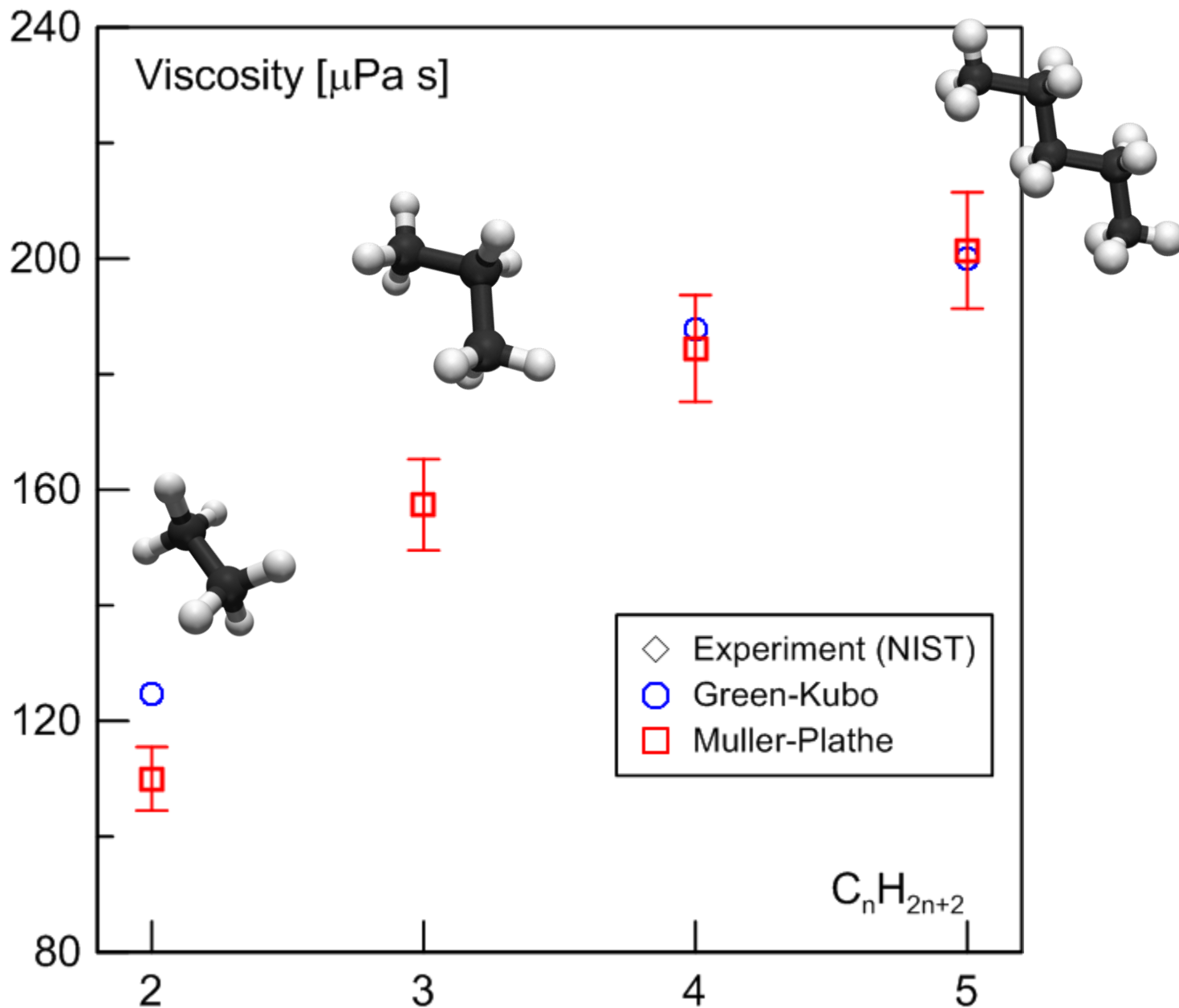
# Результаты расчета



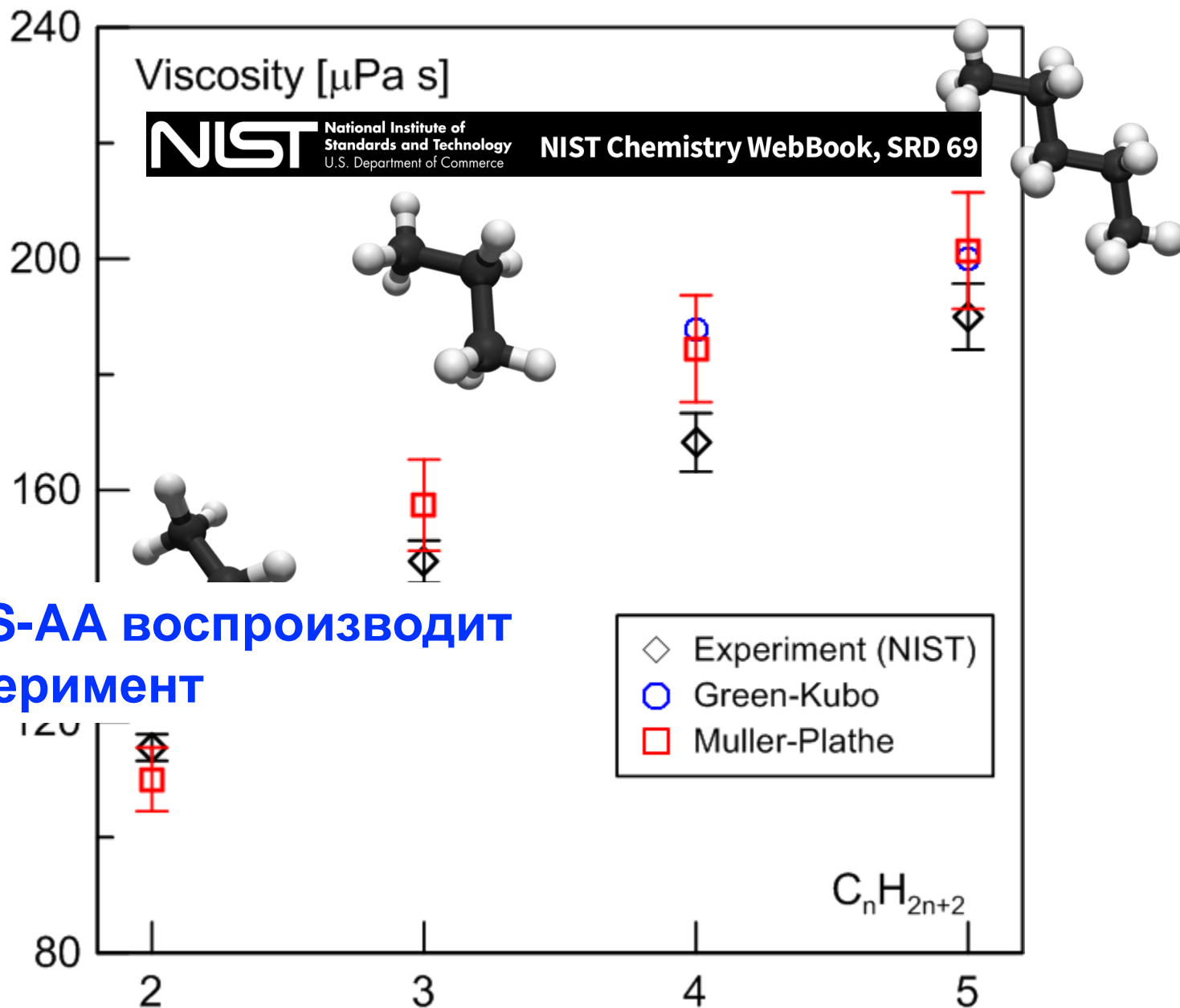


# Сравнение с экспериментом

# Результаты расчета



# Результаты расчета



**ОPLS-AA воспроизводит эксперимент**

# Выводы

Степень скорелированности **определяет сложность** расчета транспортных коэффициентов

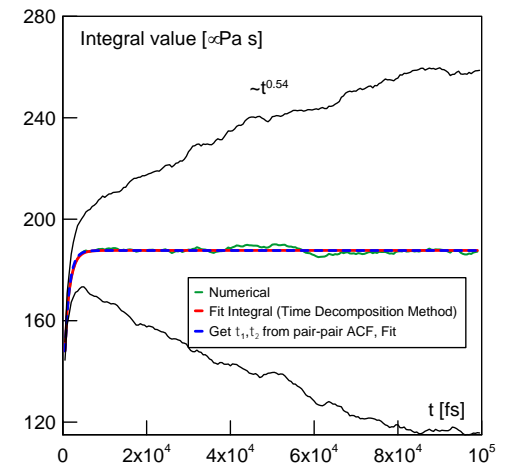
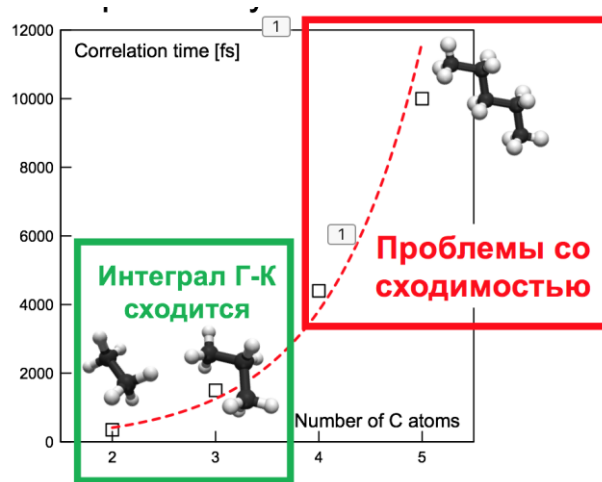
$$\int_{t=0}^{2\text{ps}} C_V(t) dt / 3$$

||

$$D = D_n + D_a$$

||

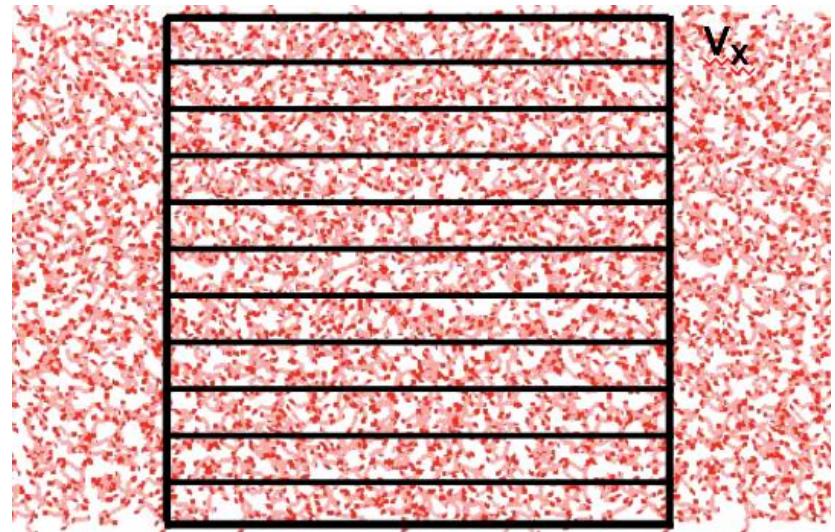
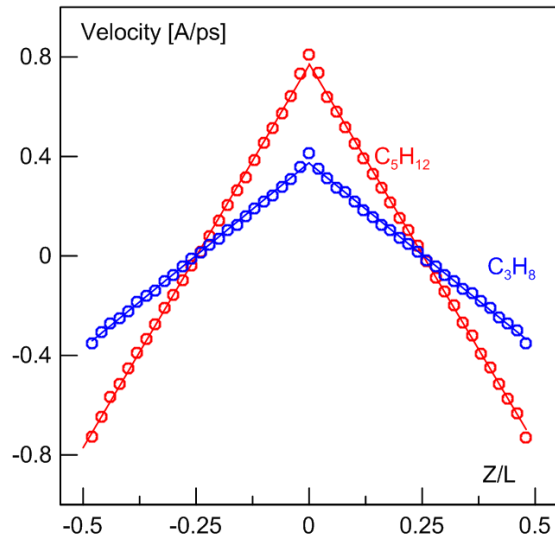
$$\int_{t=2\text{ps}}^{\infty} A t^{-\beta} dt$$



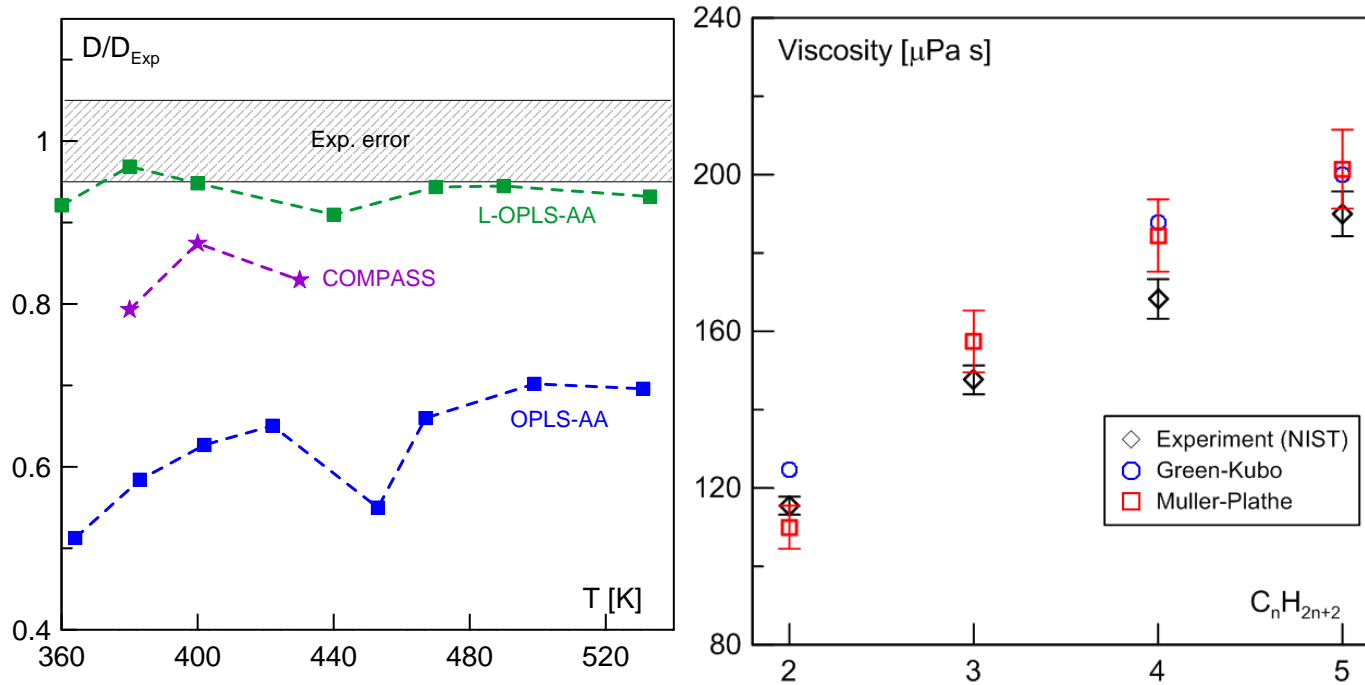
# Выводы

Степень скореллированности **определяет сложность**  
расчета транспортных коэффициентов

**Методы неравновесной МД**  
требуют на порядок меньших времен расчета



# Выводы



Полноатомные модели позволяют рассчитывать диффузию и вязкость с **хорошей точностью**

# Выводы

Степень корреляции **определяет сложность** расчета транспортных коэффициентов

**Методы неравновесной МД** требуют на порядок меньших времен расчета

Полноатомные модели позволяют рассчитывать диффузию и вязкость с **хорошей точностью**

*Работа выполнена при поддержке грантов  
РНФ 14-50-00124 и 17-79-20391,  
а также программы «5-100» НИУ ВШЭ.*

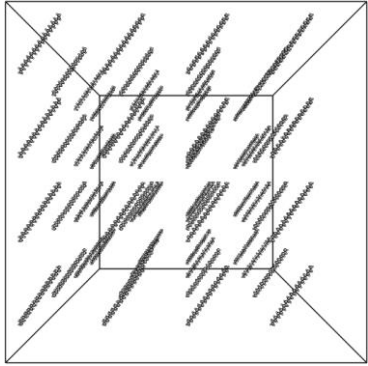






# Preparing the system

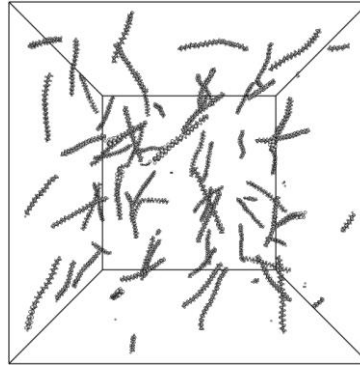
Initial configuration\*



$r_{\text{intermolec}} > r_{\text{cut}} (12 \text{ \AA})$

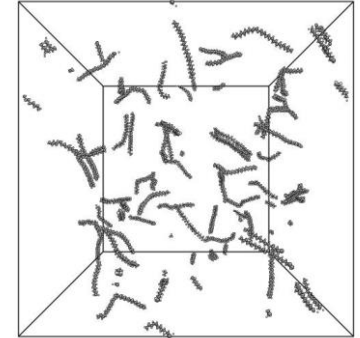
1.  
NVE  
→  
 $T = 500 \text{ K}$   
 $100 \text{ ps}$   
 $\Delta t = 1 \text{ fs}$

Gas



2.  
NVE  
→  
compression  
 $0.25 \times L$   
 $100 \text{ ps}$

Liquid



Liquid relaxation:

Parameters:

→ 3. NPT (2 ns)  $P \sim 1 \text{ atm}$ ,  $T \sim 360 \text{ K}$ ,  $\rho_{\text{ave}}$  (0.5 ns)

4. NVT (2 ns)  $\rho_{\text{ave}}$ ,  $T \sim 360 \text{ K}$

5. NVE, Warming-up (0.5 ns),  $T \sim 700 \text{ K}$

6. NVT, Cooling back to  $T \sim 360 \text{ K}$ , NVT

$\rho$ ,  $T$ ,  $P \sim 1 \text{ atm}$

$\langle K^2 \rangle$  - the relative shape anisotropy parameter

\*LAMMPS The linear momentum is zeroed in the end of the relaxation process.

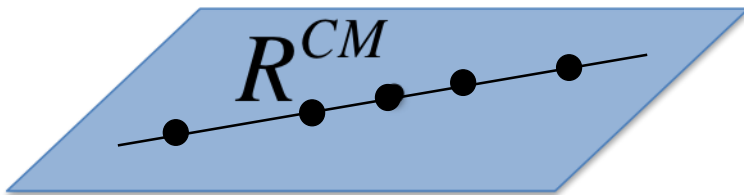
# The relative shape anisotropy $\kappa^2$

Gyration tensor

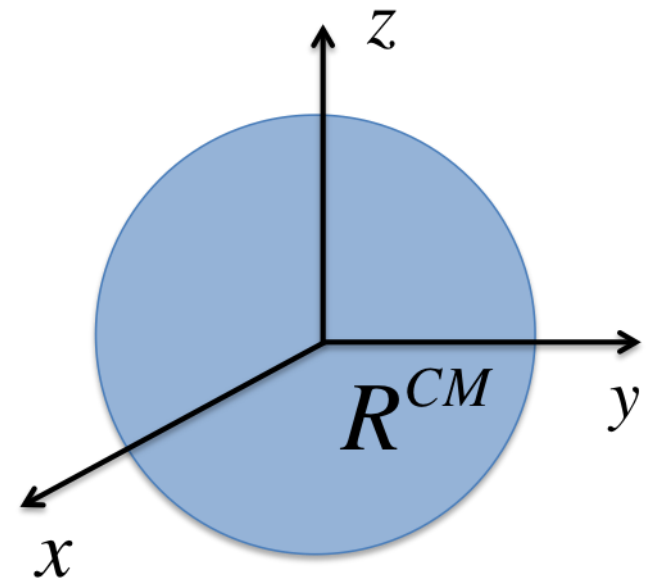
$$MS_{mn} = \sum_{i=1}^N m_i (r_m^i - R^{CM})(r_n^i - R^{CM})$$

$$\begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{zy} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} \rightarrow (\lambda_x, \lambda_y, \lambda_z) \rightarrow \kappa^2 = \frac{3}{2} \frac{\lambda_x^4 + \lambda_y^4 + \lambda_z^4}{(\lambda_x^2 + \lambda_y^2 + \lambda_z^2)^2} - \frac{1}{2}$$

eigenvalues

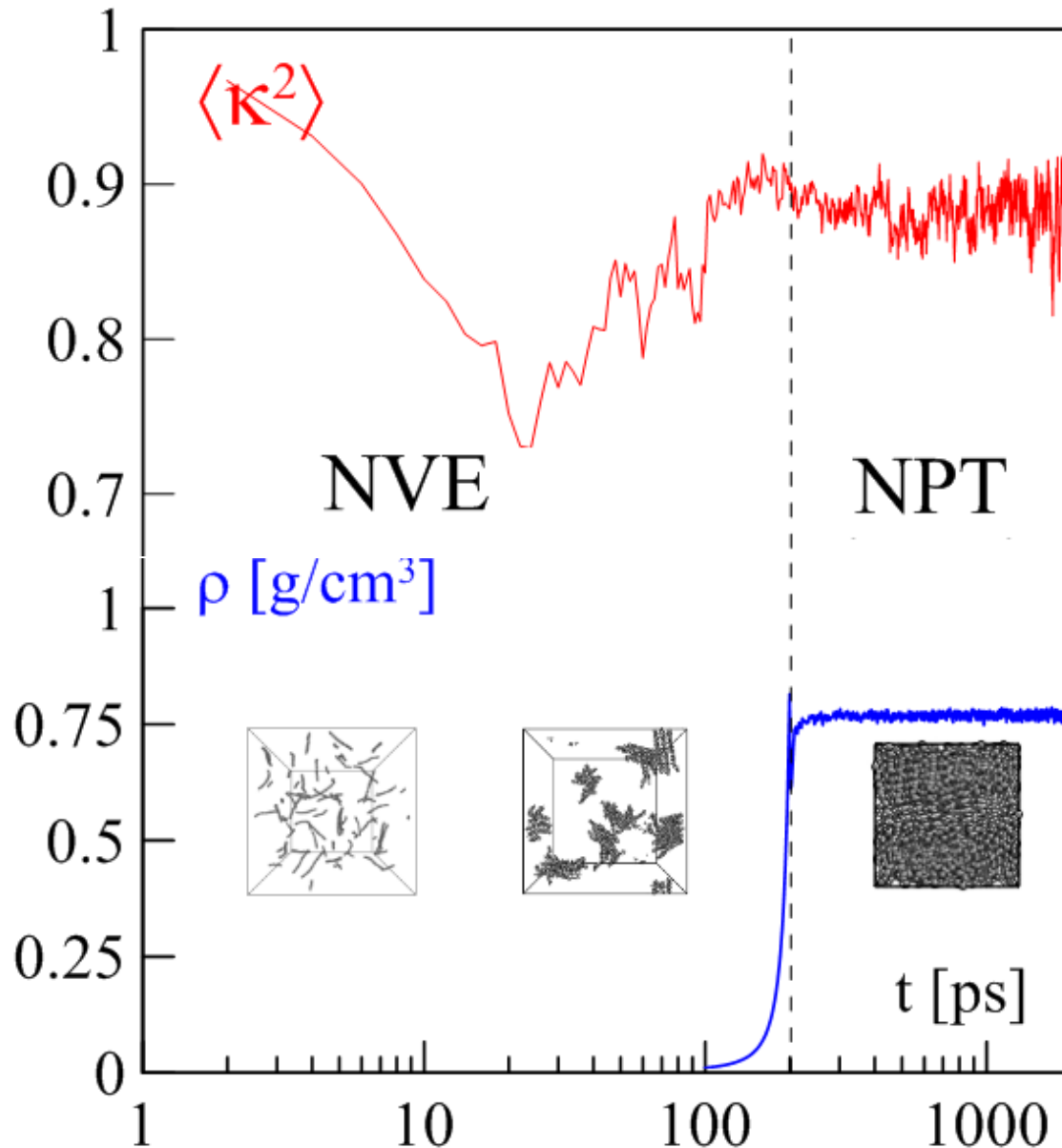


$$\kappa^2 = 1$$



$$\kappa^2 = 0$$

# The relaxation parameters



P~1 atm  
T=360 K

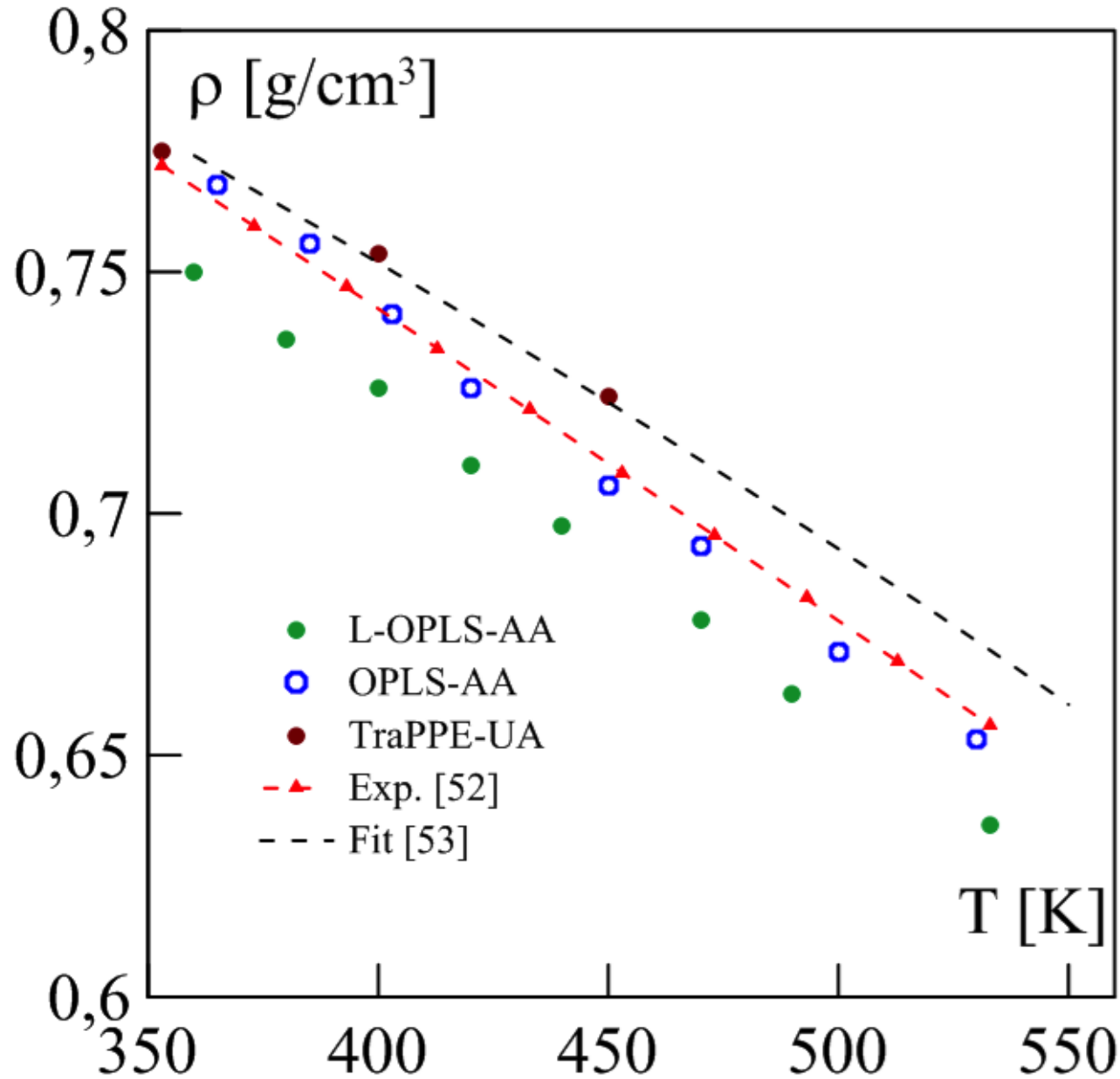
# Replication



125-**3375** molecules – AA (**330000** atoms)

125-**8000** molecules – UA (**240000** atoms)

# Systems for calculation



# How the other systems are obtained at T

1. System at 360 K
2. Calculating density (Yaws approximation)
3. Deforming the unit cell to the approximation density  
(700 K, NVE, 100 ps)
4. NVT ( $T$ ,  $\rho_{\text{approx}}$ , 500 ps)
5. NPT ( $T$ ,  $P \sim 1$  atm, 500 ps)
6. Calculating the average density during the process
7. Deforming the unit cell to the average density  
( $T$ , NVE, 100 ps)
8. Zeroing the linear momentum of the system
9. Scaling the velocity distribution to T

# Size effects in the E-S method

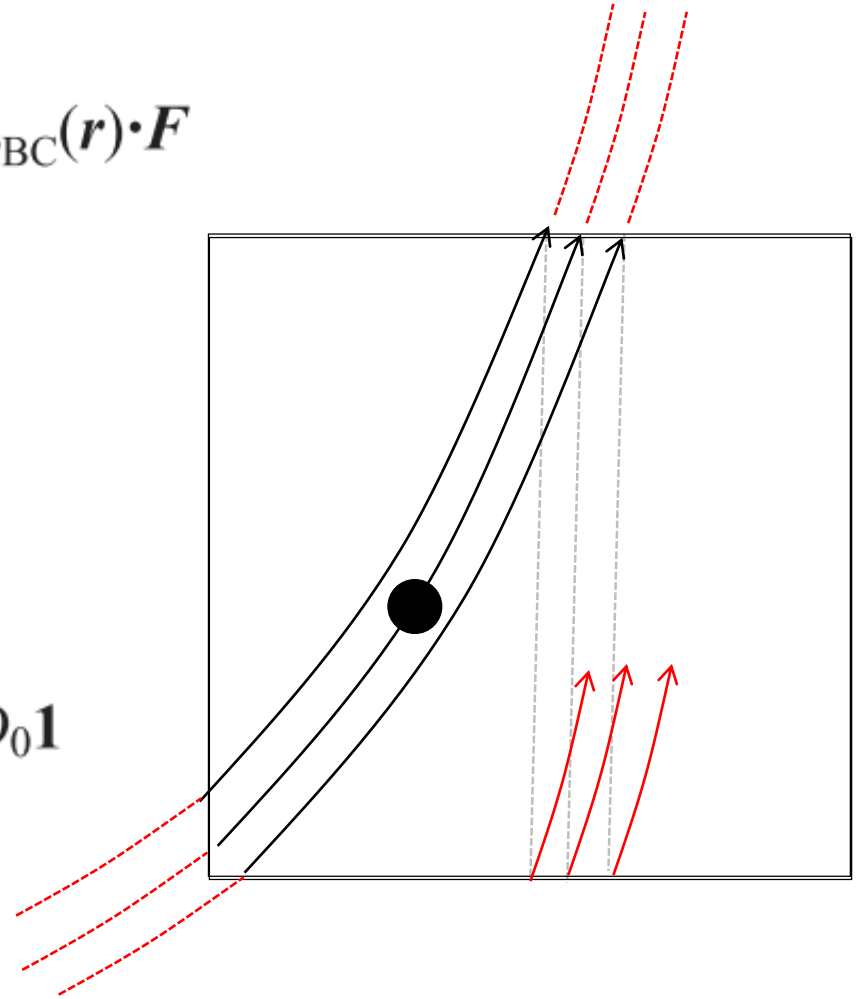
$$v(\mathbf{r}) = \mathbf{T}_{\text{PBC}}(\mathbf{r}) \cdot \mathbf{F}$$

For **Periodic Boundary Conditions**:

$$\mathbf{T}_{\text{PBC}}(\mathbf{r}) = \sum_{\substack{\mathbf{k} \\ \mathbf{k} \neq 0}} \frac{\exp(-i\mathbf{k} \cdot \mathbf{r})}{\eta k^2 V} \left( \mathbf{1} - \frac{\mathbf{k}\mathbf{k}}{k^2} \right)$$

For infinite system:

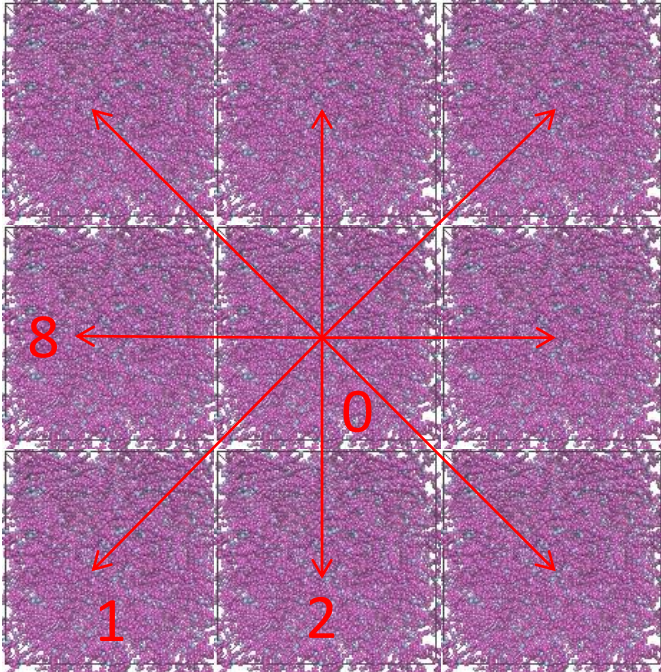
$$\mathbf{T}_0(\mathbf{r}) = \frac{1}{8\pi\eta r} \left( \mathbf{1} + \frac{\mathbf{r}\mathbf{r}}{r^2} \right) = (k_B T)^{-1} D_0 \mathbf{1}$$



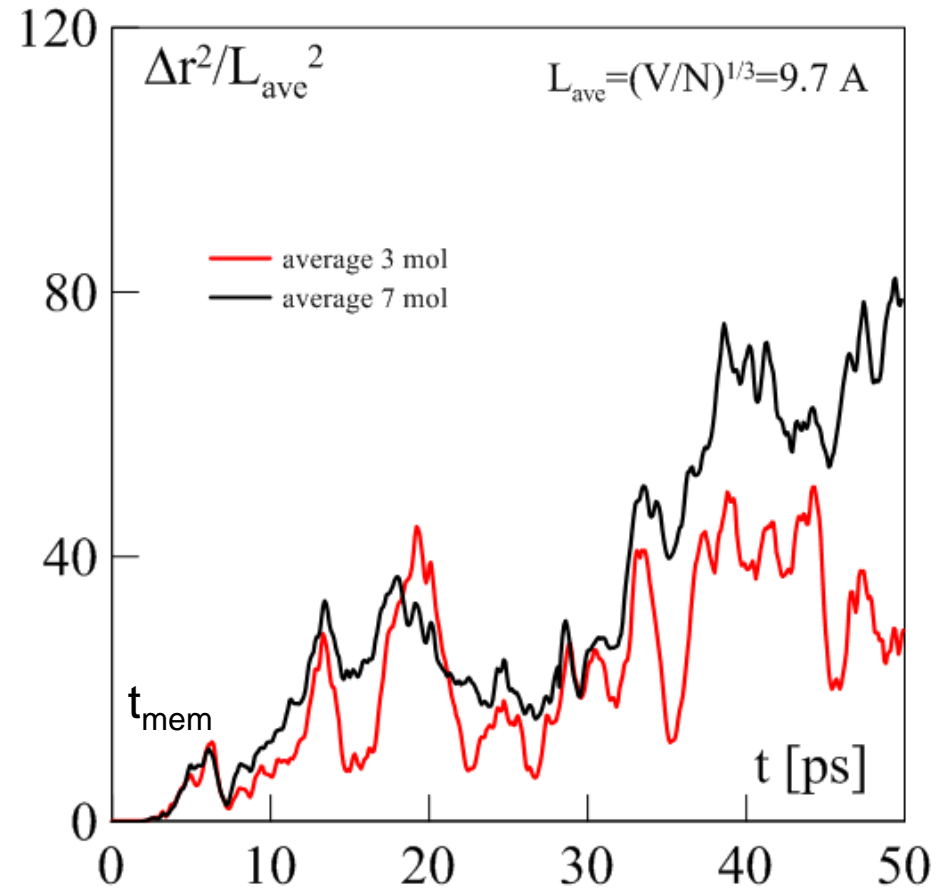
$$\mathbf{D}_{\text{PBC}} = D_0 \mathbf{1} + k_B T \lim_{r \rightarrow 0} [\mathbf{T}_{\text{PBC}}(\mathbf{r}) - \mathbf{T}_0(\mathbf{r})]$$



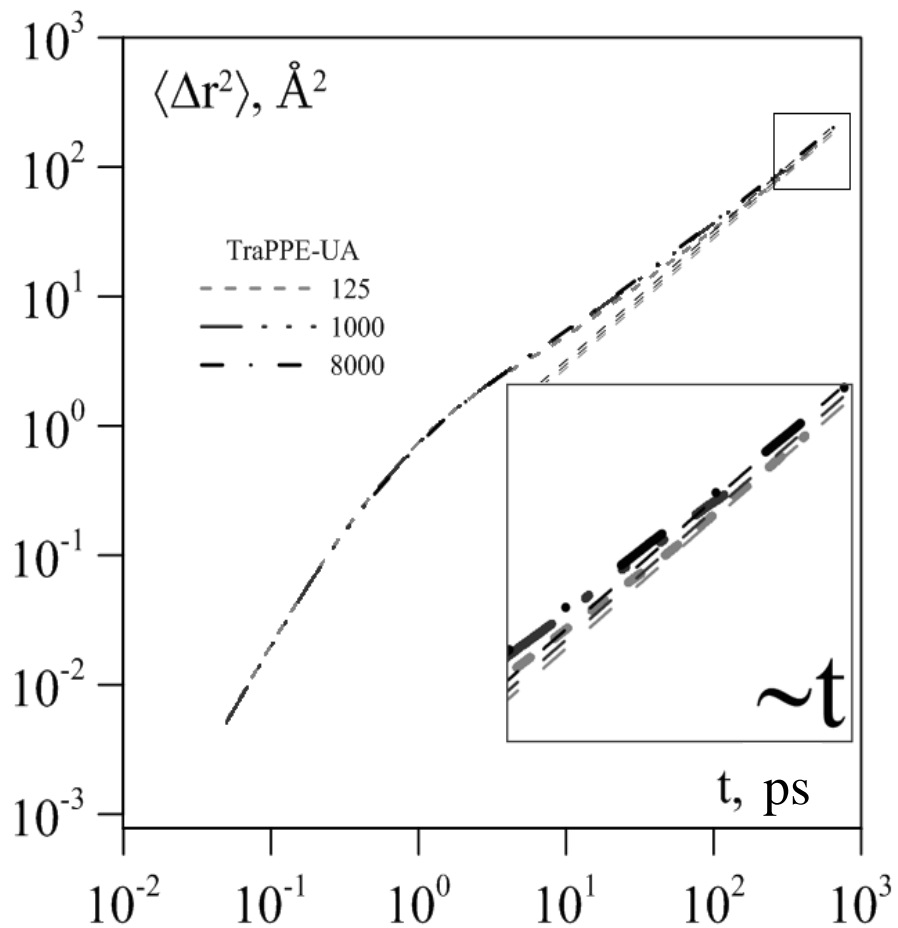
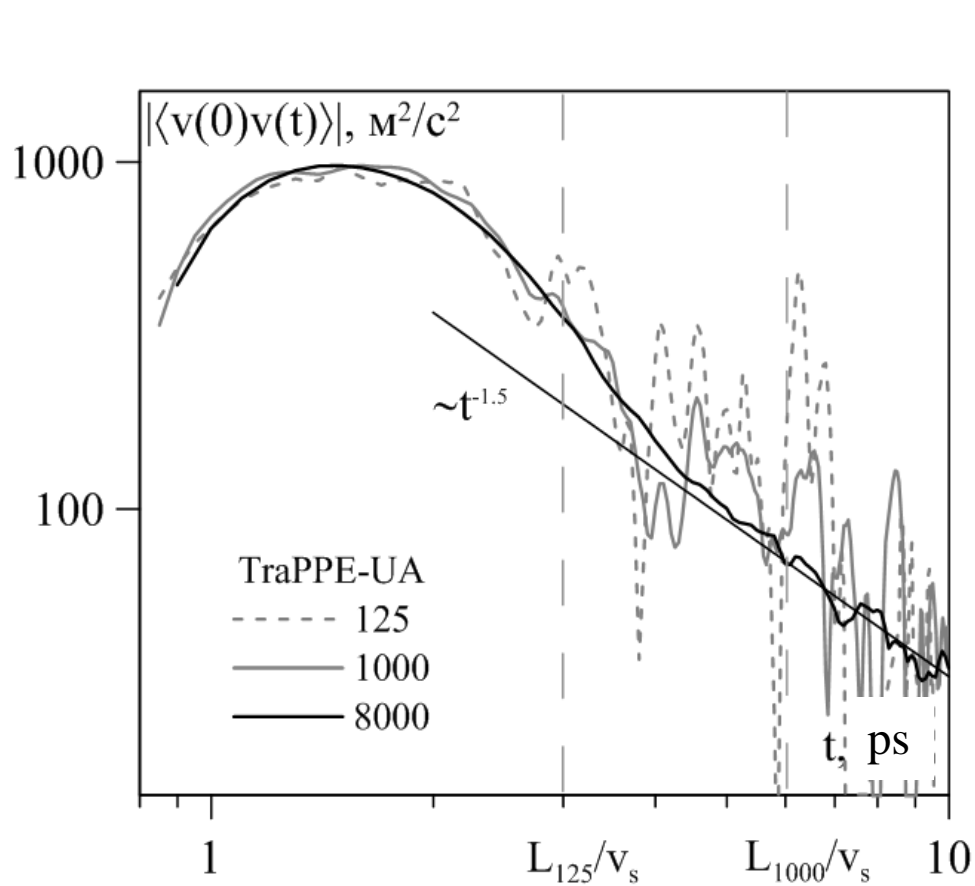
# Replication independency



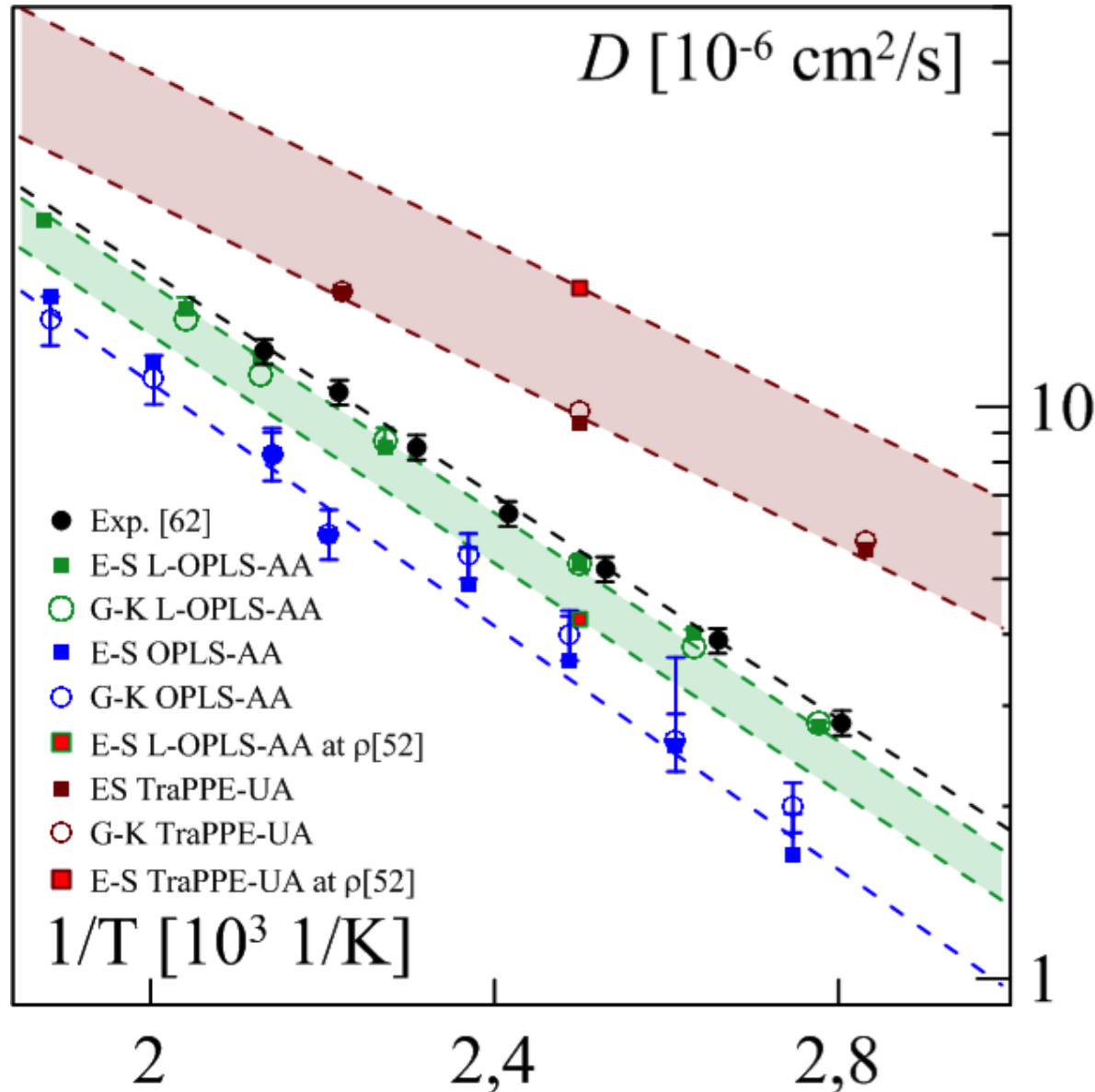
$$\Delta r_{\text{mol}}^2 = \sum_{k=1}^{26} (\mathbf{r}_{\text{mol}}^k - \mathbf{r}_{\text{mol}}^0)^2$$



# The dependence on the number of molecules



# Simulation results vs experimental data



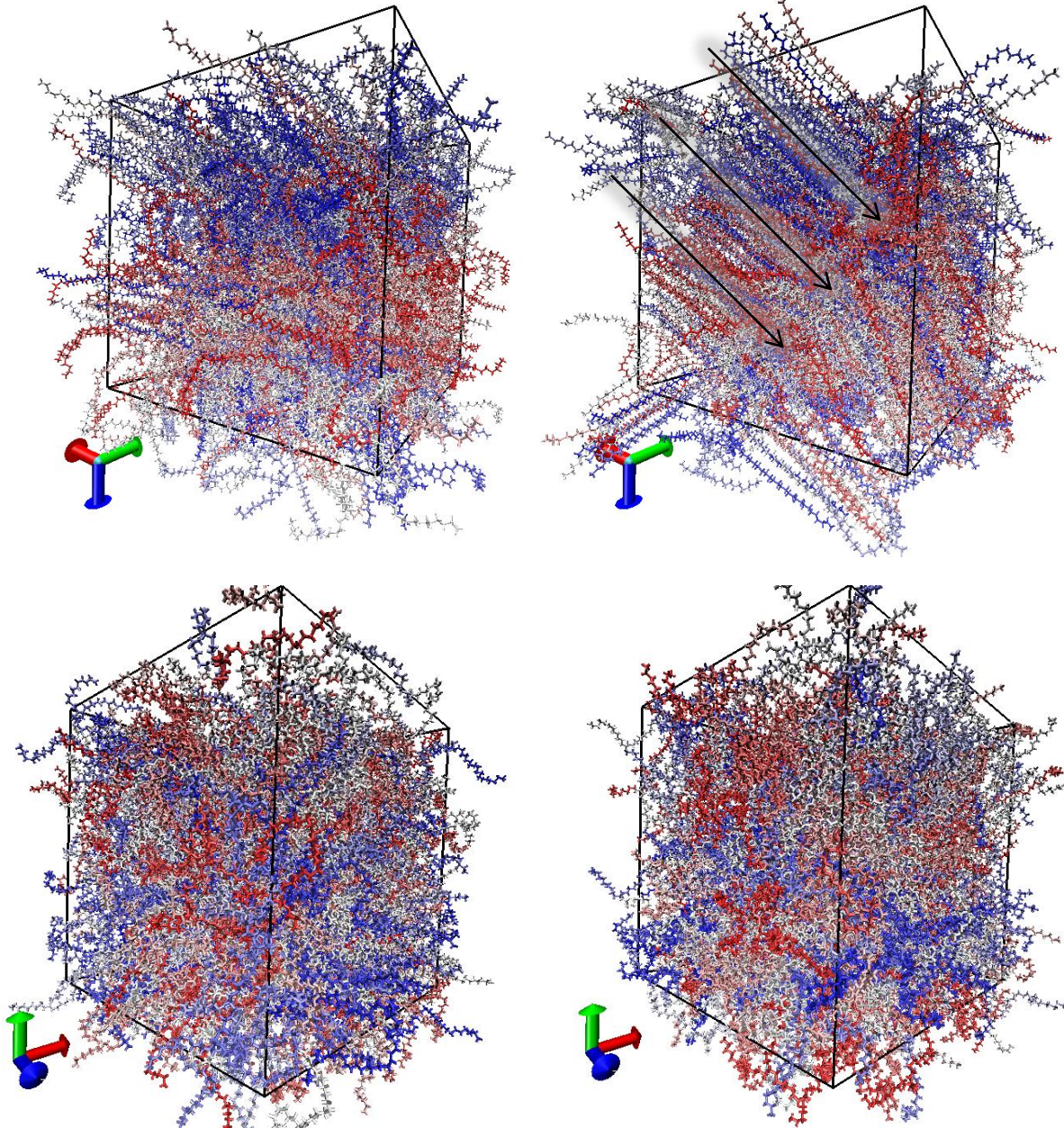
# Metastable liquid in OPLS-AA

OPLS-AA

solidifies after 1.5 ns

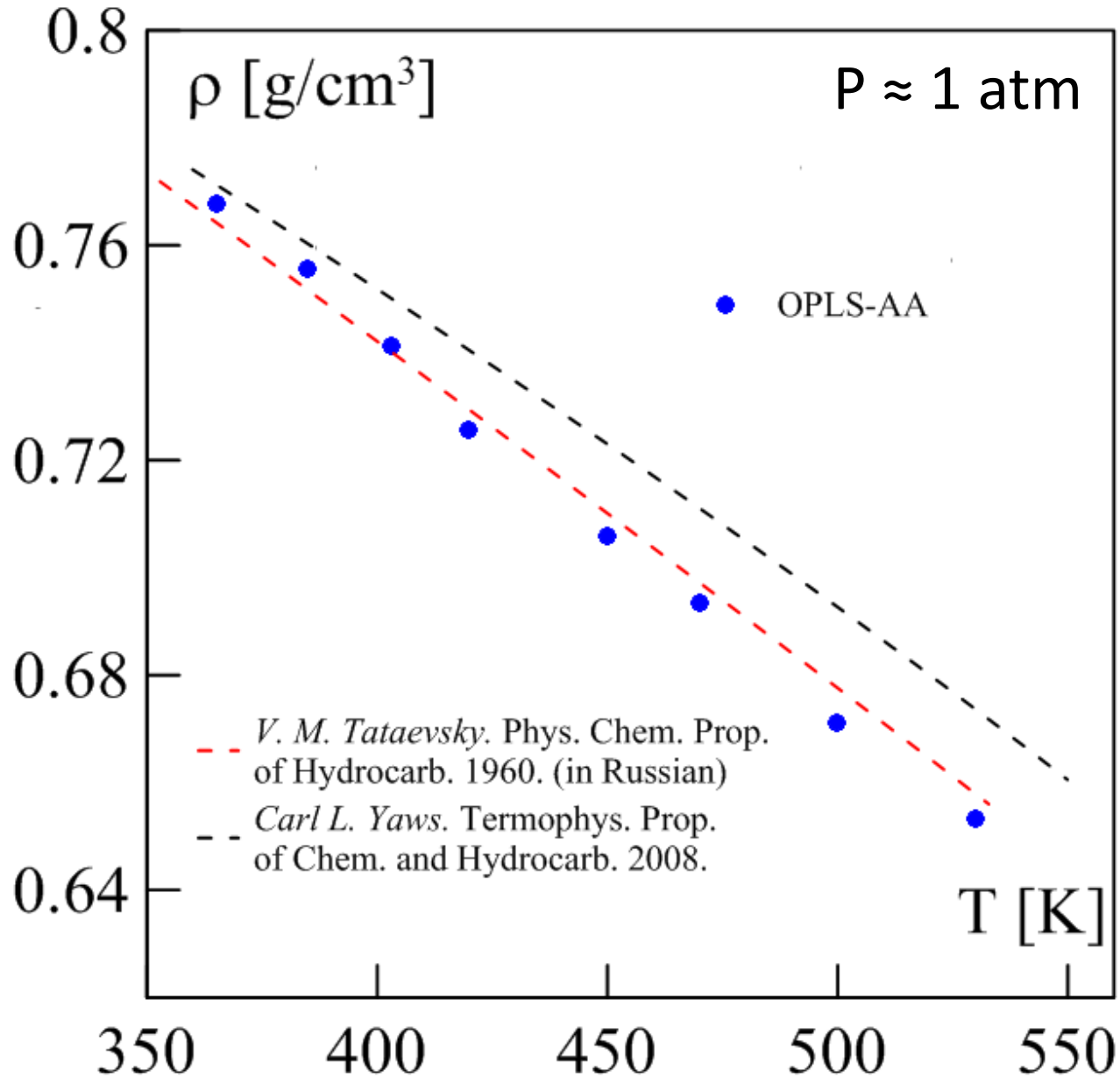
L-OPLS-AA

no solidification after 1.5 ns



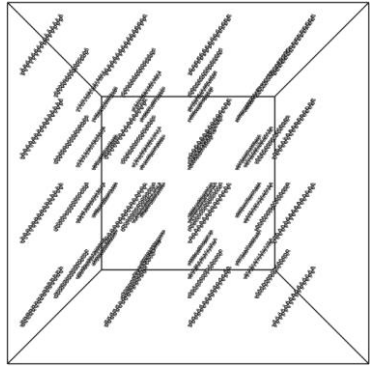


# Systems for calculation



# Preparing the system

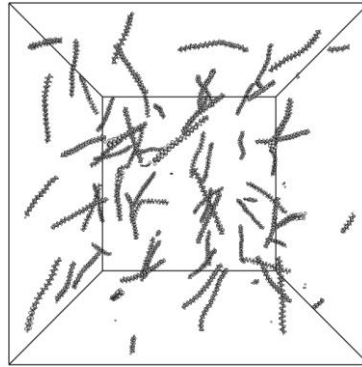
Initial configuration\*



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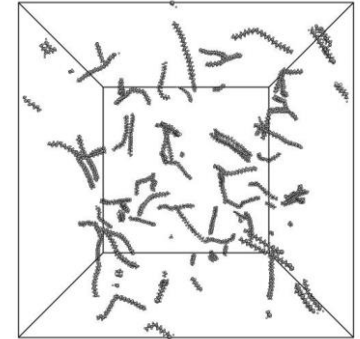
1.  
NVE  
→  
T = 500 K  
100 ps  
 $\Delta t = 1 \text{ fs}$

Gas



2.  
NVE  
→  
compression  
0.25 x L  
100 ps

Liquid



Liquid relaxation:

Parameters:

→ 3. NPT (2 ns) P~1 atm, T~360 K,  $\rho_{\text{ave}}$  (0.5 ns)

4. NVT (2 ns)  $\rho_{\text{ave}}$ , T~360 K

5. NVE, Warming-up (0.5 ns), T~700 K

6. NVT, Cooling back to T~360 K, NVT

$\rho, T, P \sim 1 \text{ atm}$

$\langle K^2 \rangle$  - the relative shape anisotropy parameter

\*LAMMPS The linear momentum is zeroed in the end of the relaxation process.

# How the other systems are obtained at T

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# The idea of molecular dynamics

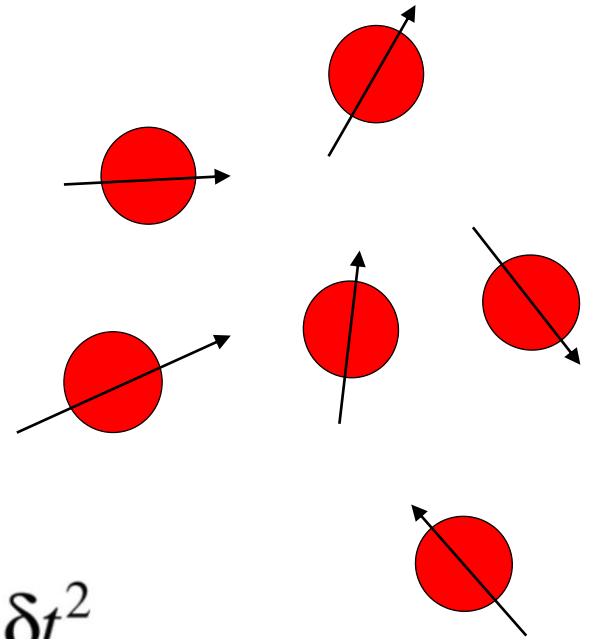
Forces between different particles are derived from some analytical force model,

$$\mathbf{F} = -\nabla V(r)$$

$$\mathbf{F}_i = \sum_{j \neq i} \mathbf{F}_{ij}$$

Equations of motion are solved with a finite difference algorithm,

$$\vec{r}_i(t + \delta t) = \vec{r}_i(t) + \vec{v}_i(t)\delta t + \frac{1}{2}\vec{a}_i(t)\delta t^2$$





T [K]	$\rho$ [g/cm <sup>3</sup> ]	Diffusion coefficient, 10 <sup>-6</sup> cm <sup>2</sup> /s		$\beta$ , $\Delta\beta=0.1$
		E-S	G-K	
364	0.768	1.7 ± 0.3	2.0 ± 0.2	2.05
383	0.756	2.5 ± 1.1	2.6 ± 0.3	2.0
402	0.741	3.6 ± 0.7	4.0 ± 0.4	1.97
422	0.726	4.9 ± 0.8	5.5 ± 0.5	1.97
453	0.706	5.9	6.0 ± 0.6	1.7
467	0.693	8.3 ± 0.9	8.2 ± 0.8	1.6
499	0.671	12.0	11.2 ± 1.1	1.5
530	0.653	15.6	14.2 ± 1.4	1.45

1. The difference between G-K and E-S is removed
2.  $\beta$  values are validated

What is the nature  
of the  $\beta$  values?

# Decay of the Velocity Autocorrelation Function\*

B. J. Alder and T. E. Wainwright

*Lawrence Radiation Laboratory, University of California, Livermore, California 94550*

(Received 10 July 1969)

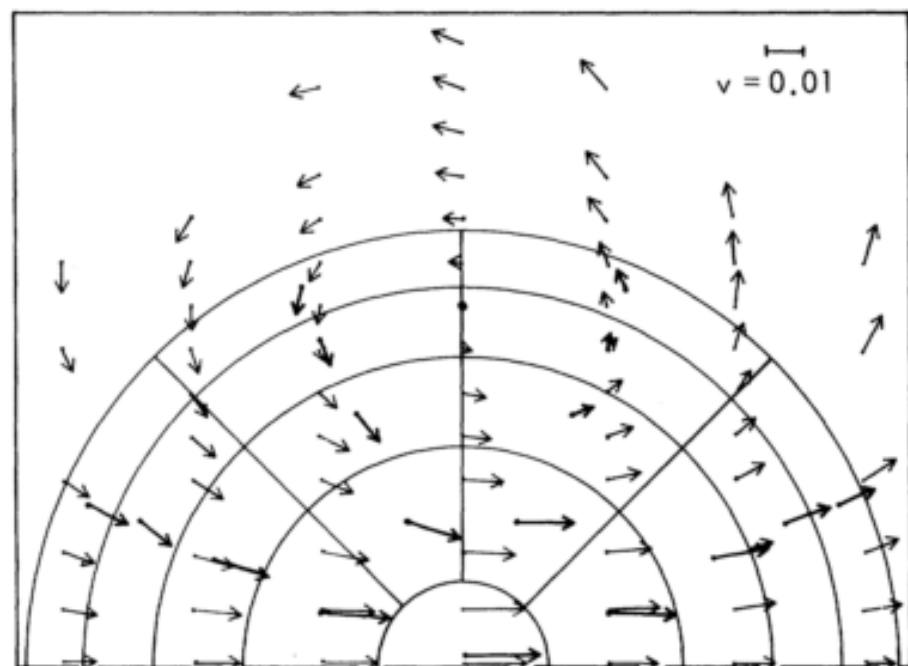


FIG. 1. Statistically averaged velocity field around a central disk from molecular dynamics (heavy arrows) compared to that given by the hydrodynamic model (light arrows). Because of symmetry only half the plane is shown. The scale of distance is indicated by the size of the central disk as shown by the smallest half-circle.

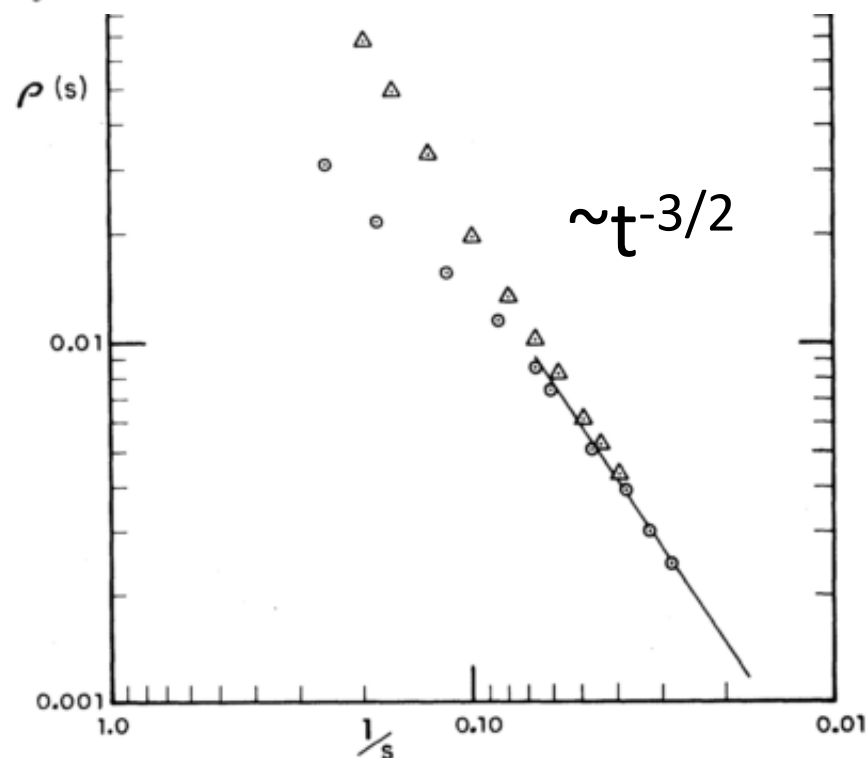
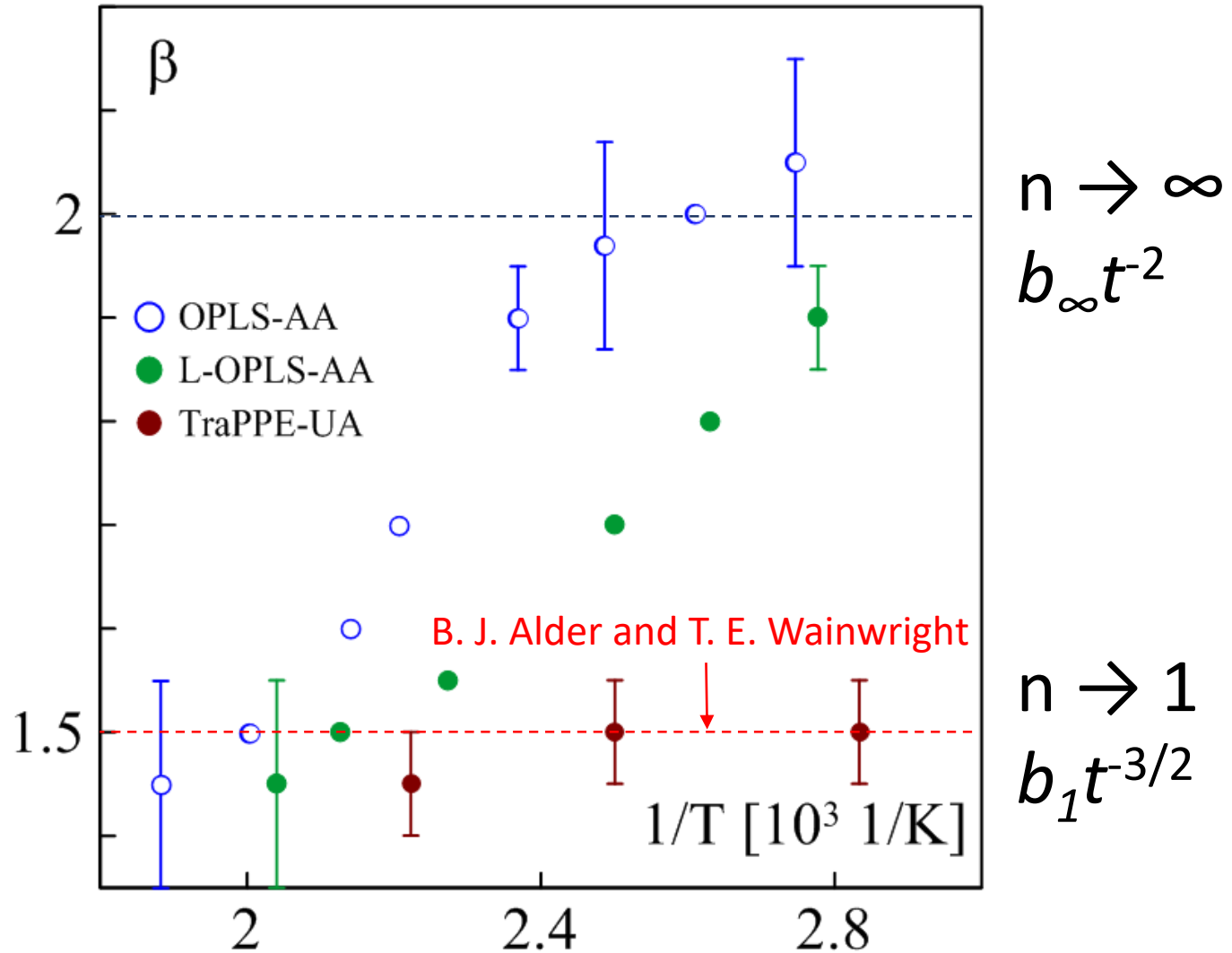


FIG. 2. Comparison of the velocity autocorrelation function  $\rho(s)$  as a function of time (in terms of mean collision times  $s$ ) between the hydrodynamic model (circles) and a 500-hard-sphere molecular-dynamic calculation (triangles) at a volume relative to close packing of 3 on a log-log plot. The straight line is drawn with a slope corresponding to  $s^{-3/2}$ . To the molecular dynamics  $\rho(s)$

# Pomeau VACF series expansion\*

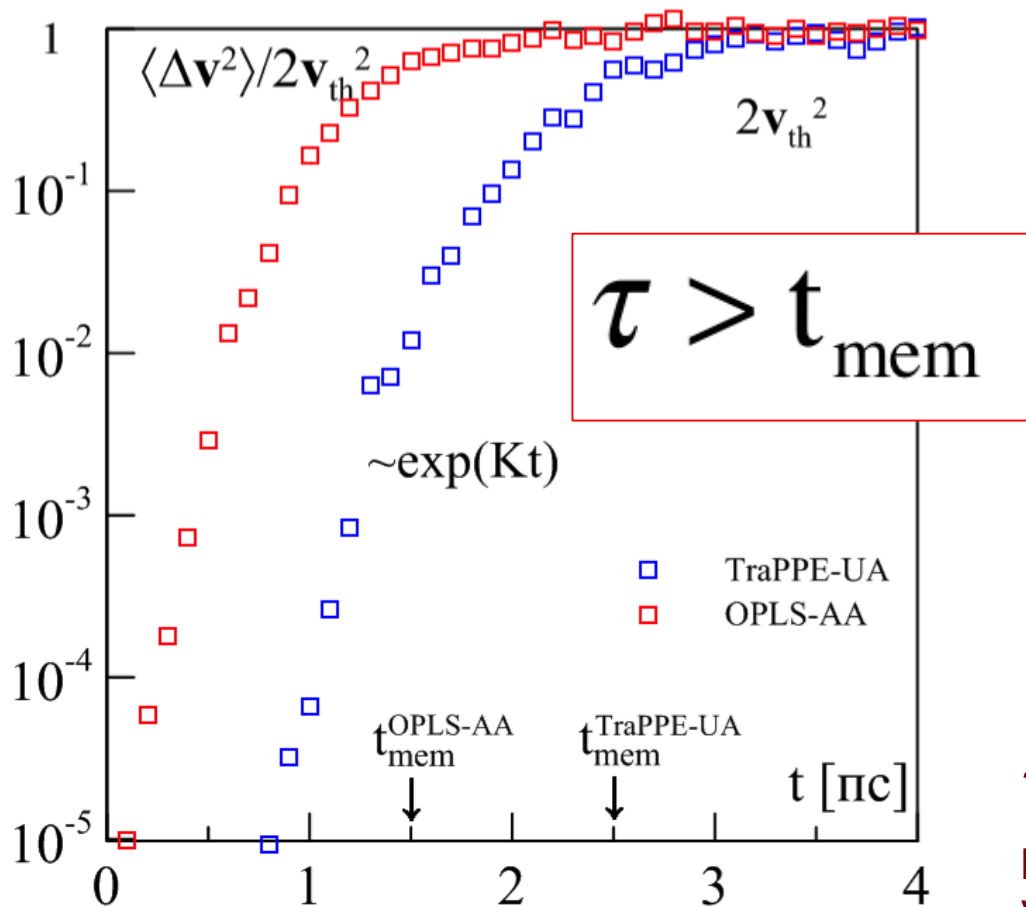
$$C_v(t) \simeq \sum_{n=1}^{\infty} b_n t^{1/2^n - 2}$$



\*Pomeau Y. // Phys. Rev. A. 1973. V. 7. № 3. P. 1134-1147.

# Determination of $\tau^*$

The same initial conditions  $\begin{cases} \rightarrow (\mathbf{r}', \mathbf{v}') - \text{MD trajectory integrated with 1 fs;} \\ \rightarrow (\mathbf{r}'', \mathbf{v}'') - \text{MD trajectory integrated with 0.1 fs;} \end{cases}$



Lyapunov instability:

$$\langle \Delta \mathbf{v}^2 \rangle = \sum_{mol} (v_i' - v_i'')^2 / N$$

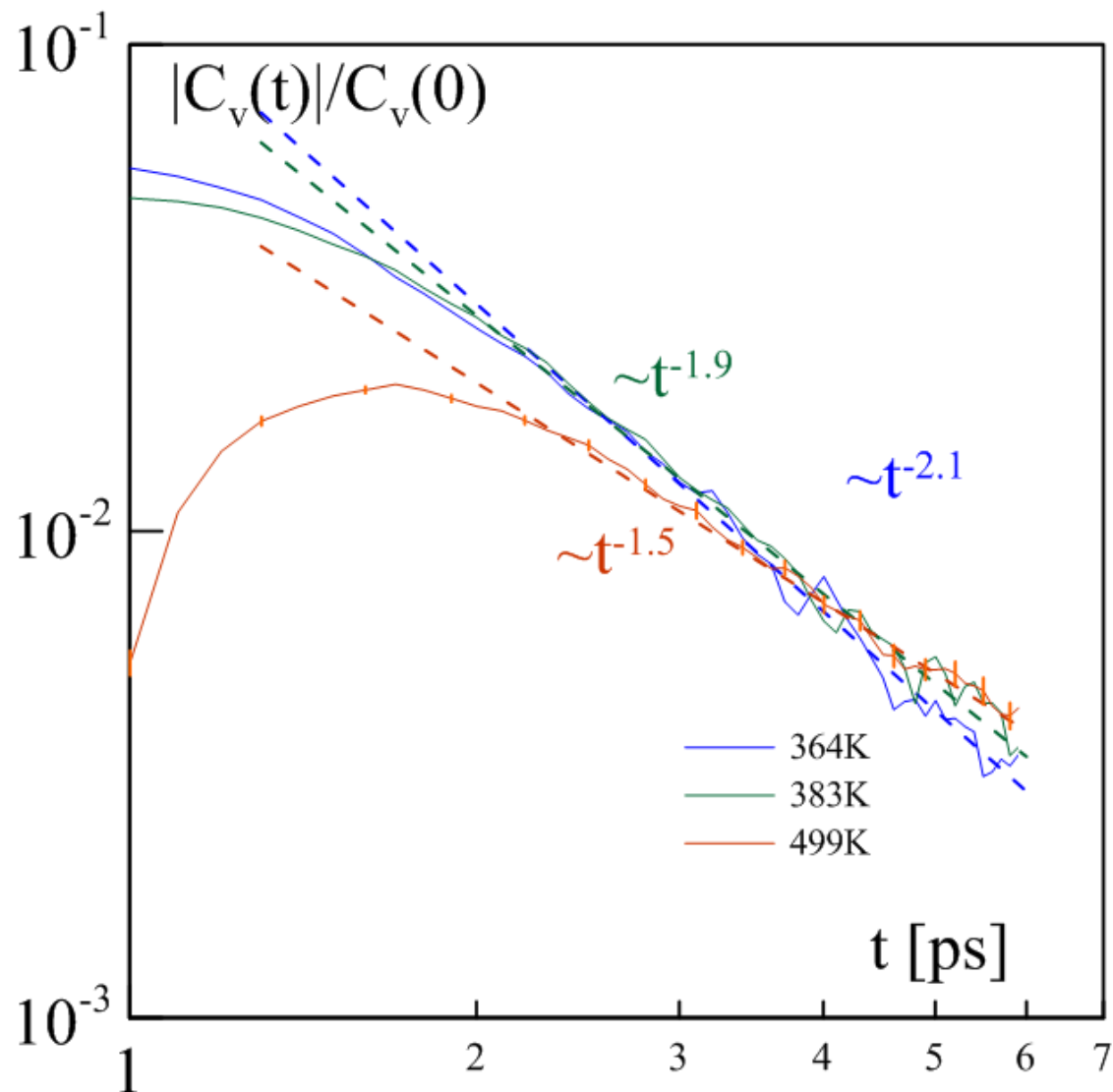
$$\langle \Delta \mathbf{v}^2 \rangle \sim \exp(Kt), \quad t < t_{mem}$$

$$\langle \Delta \mathbf{v}^2 \rangle = 2v_{th}^2, \quad t > t_{mem}$$

$$v_{th}^2 = 3k_B T / m$$

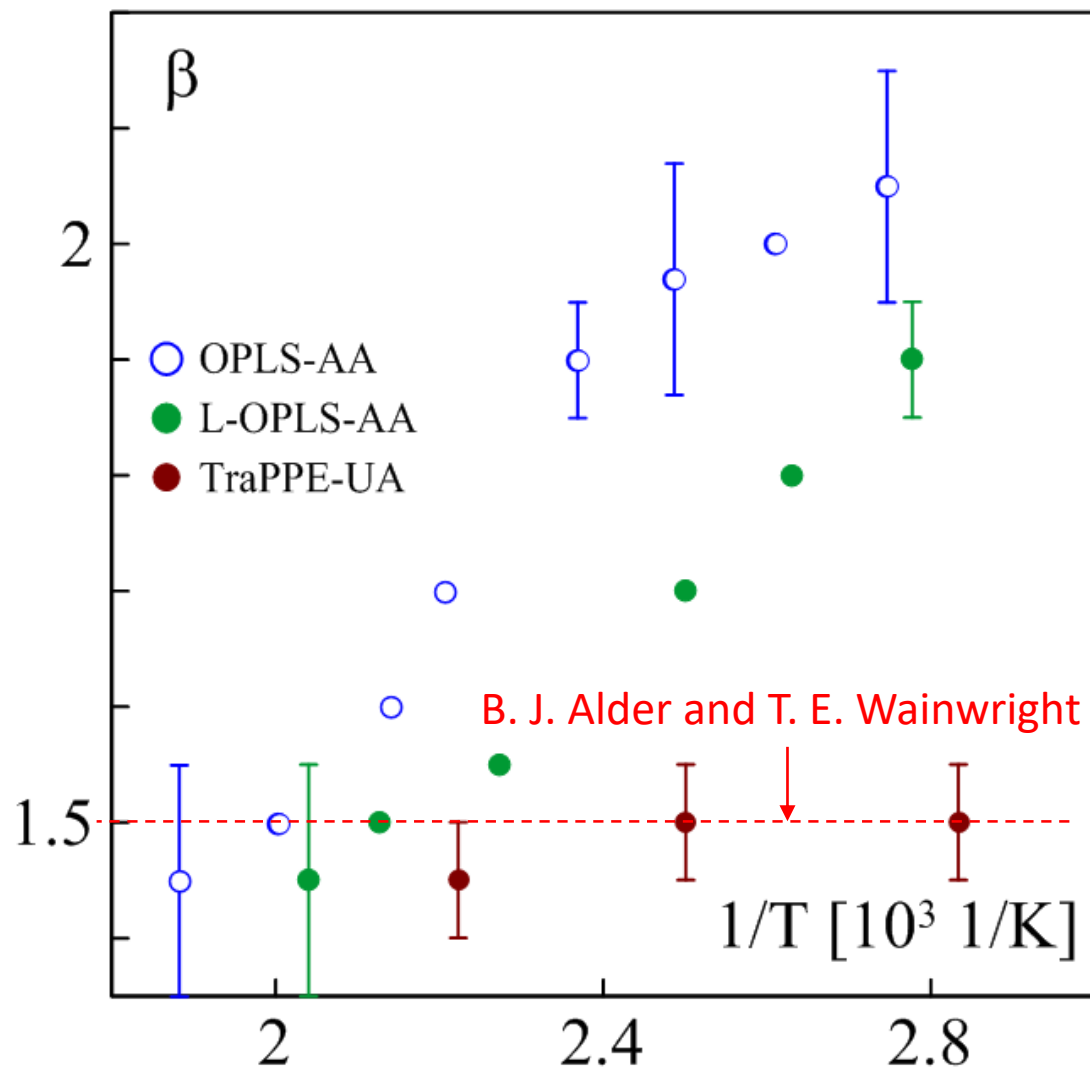
\* G. E. Norman, V. V. Stegailov // Math. Mod. Comput. Simulat. 2013, Vol. 5, No. 4, pp. 305–333.

# The $C_v$ asymptotes for n-triacontane

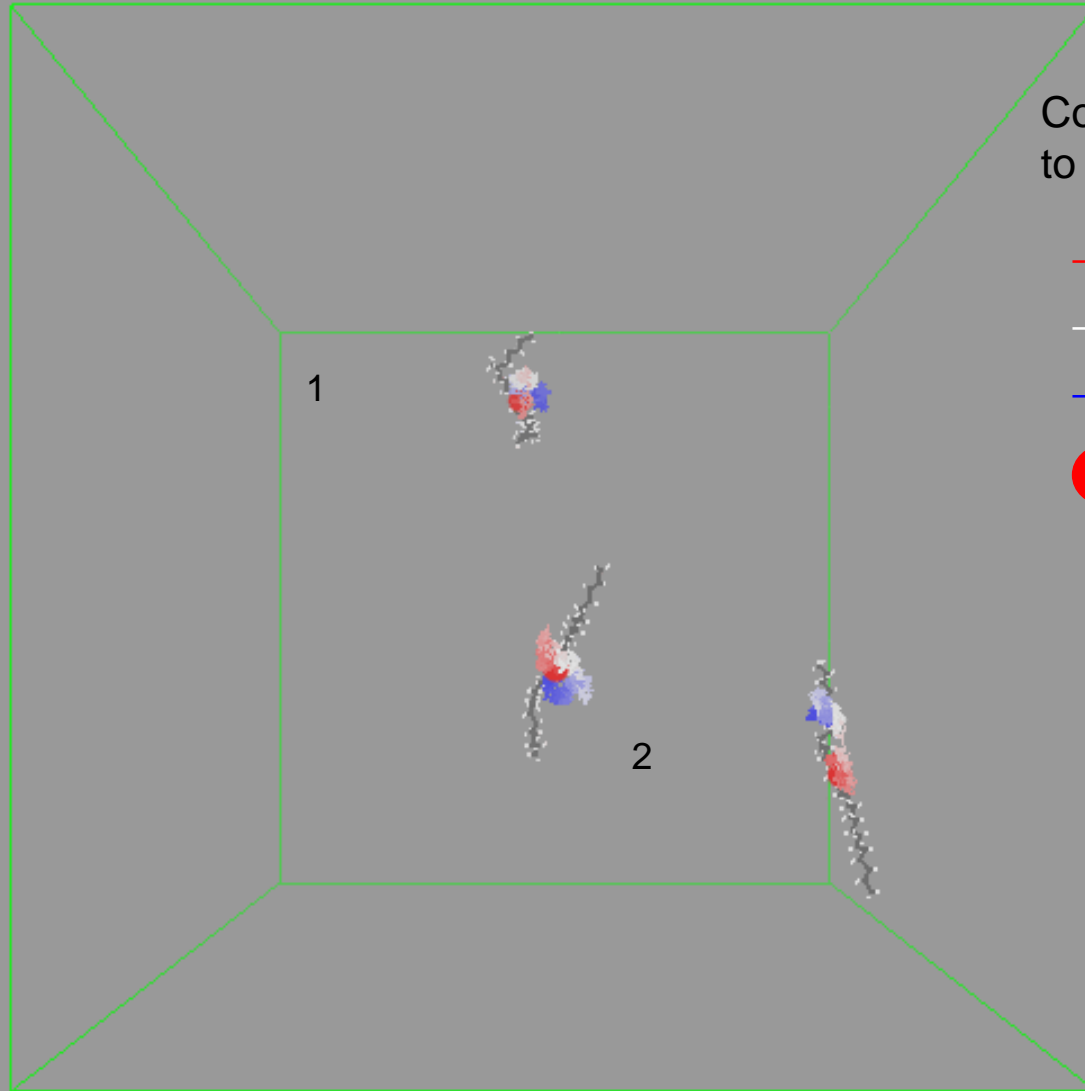


# $\beta$ dependence on temperature

$$C_V \sim t^{-\beta}$$

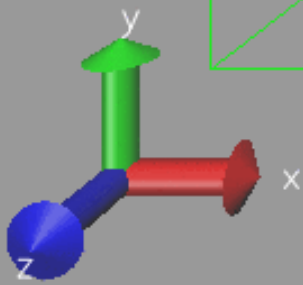


# 1 ns MD trajectory



Coloring corresponds to the current time:

- start
- middle
- end
- molecule COM





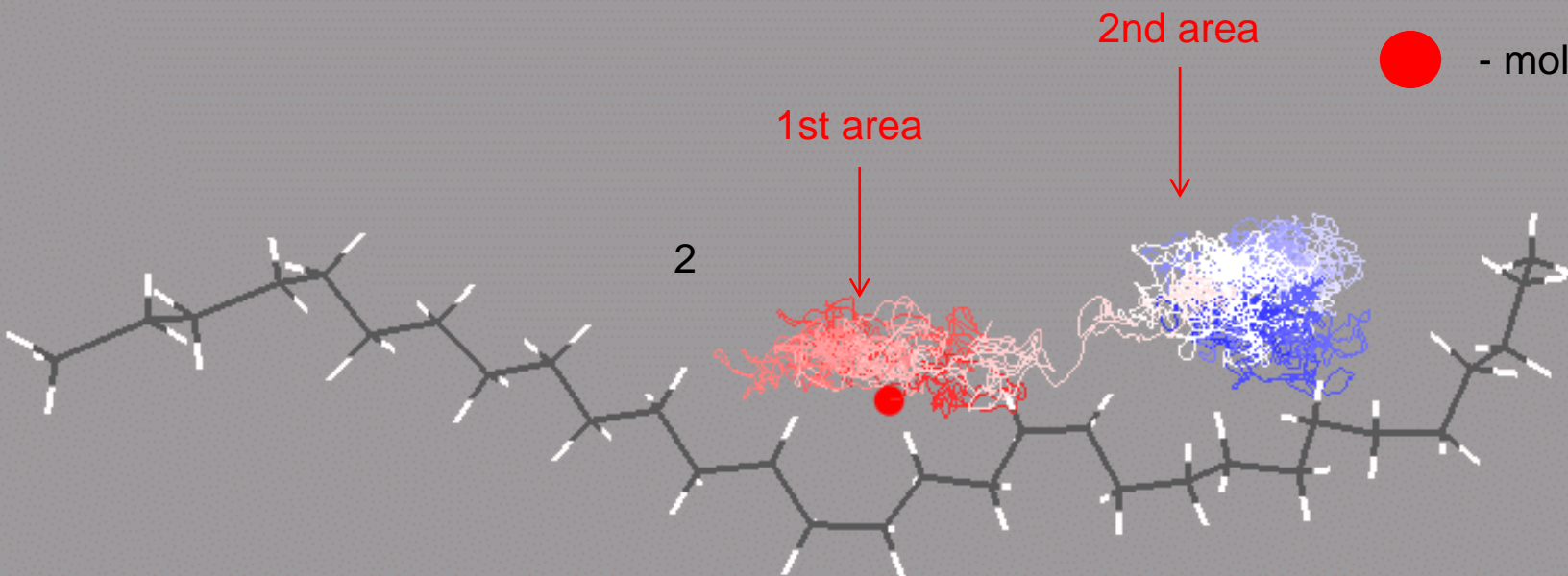
Coloring corresponds to the current time:

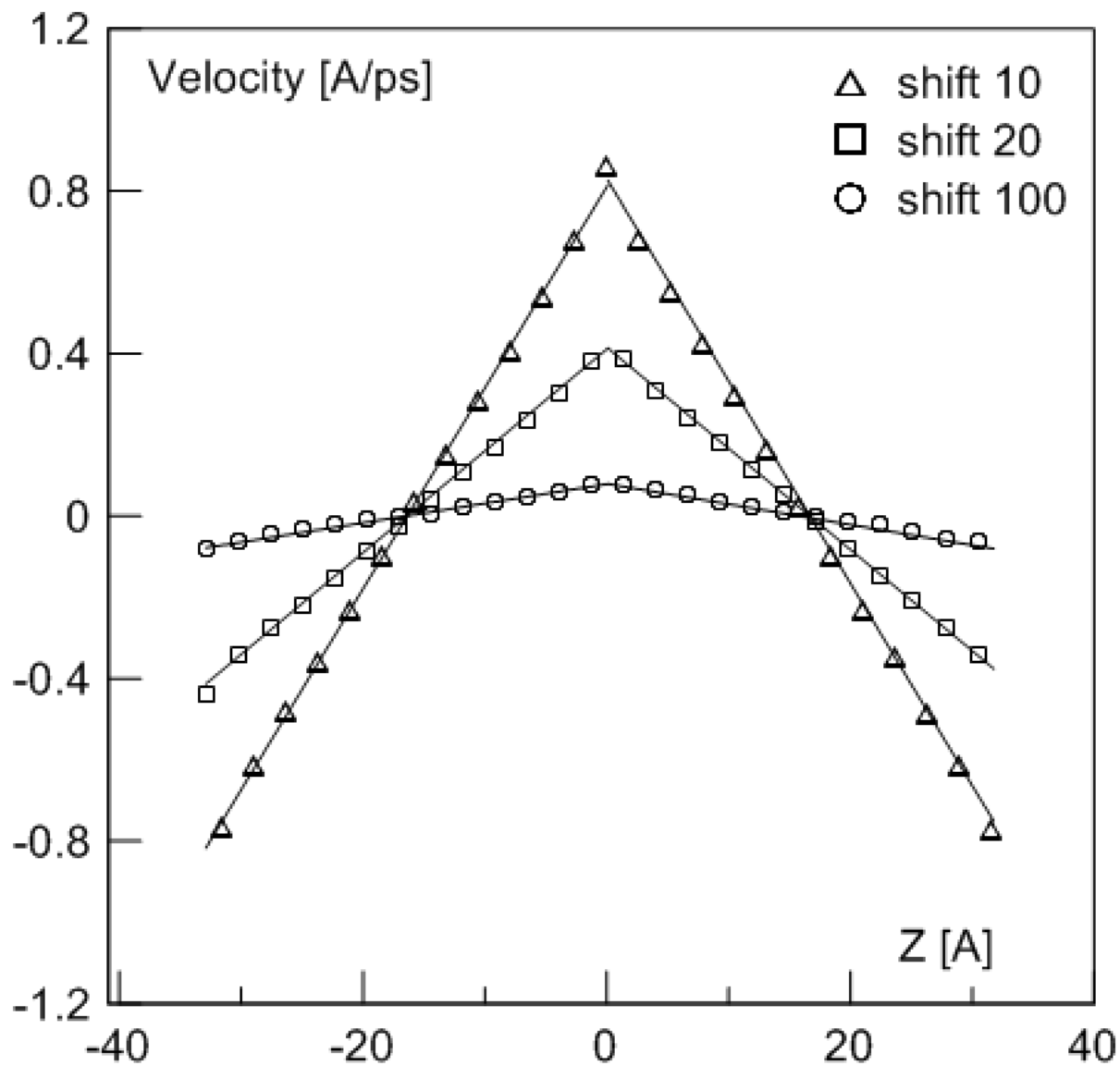
— - start

— - middle

— - end

● - molecule COM





# Техника усреднения

$$\langle \Delta r^2 \rangle = 6Dt$$

The 1<sup>st</sup> stage of the averaging:

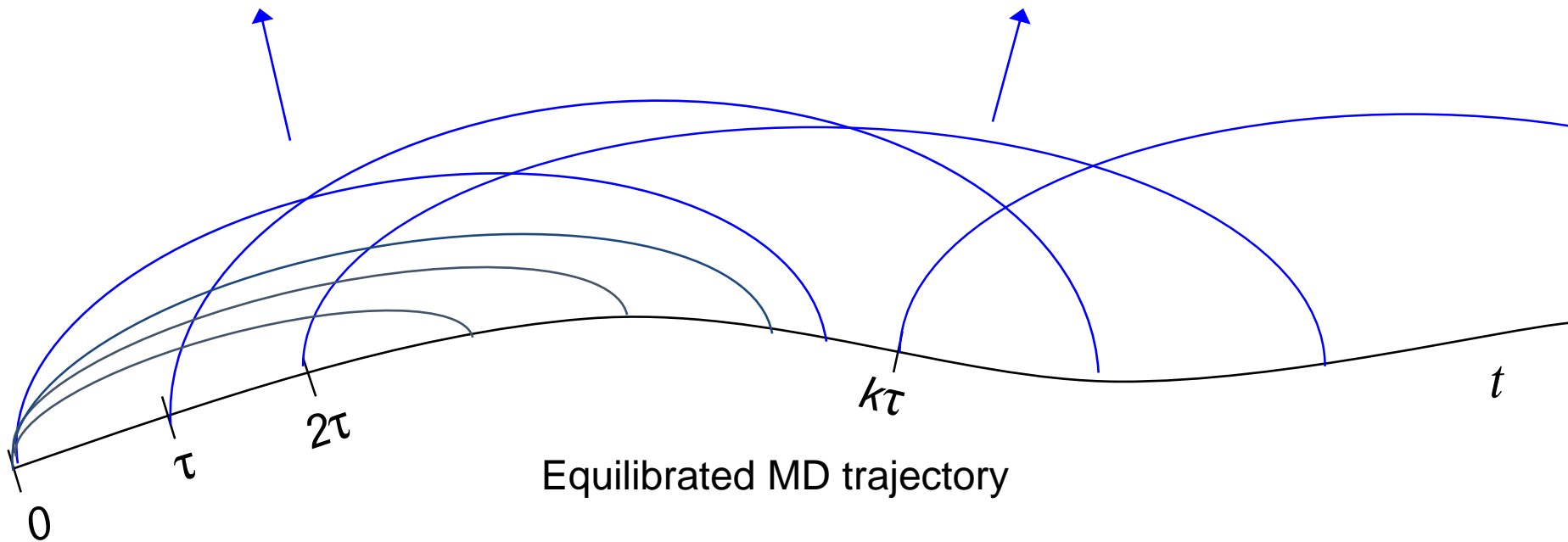
$$\langle \Delta r^2 \rangle = \sum_N (r_i^{CM}(t) - r_i^{CM}(0))^2 / N$$

$N$  - Number of the molecules (8000)

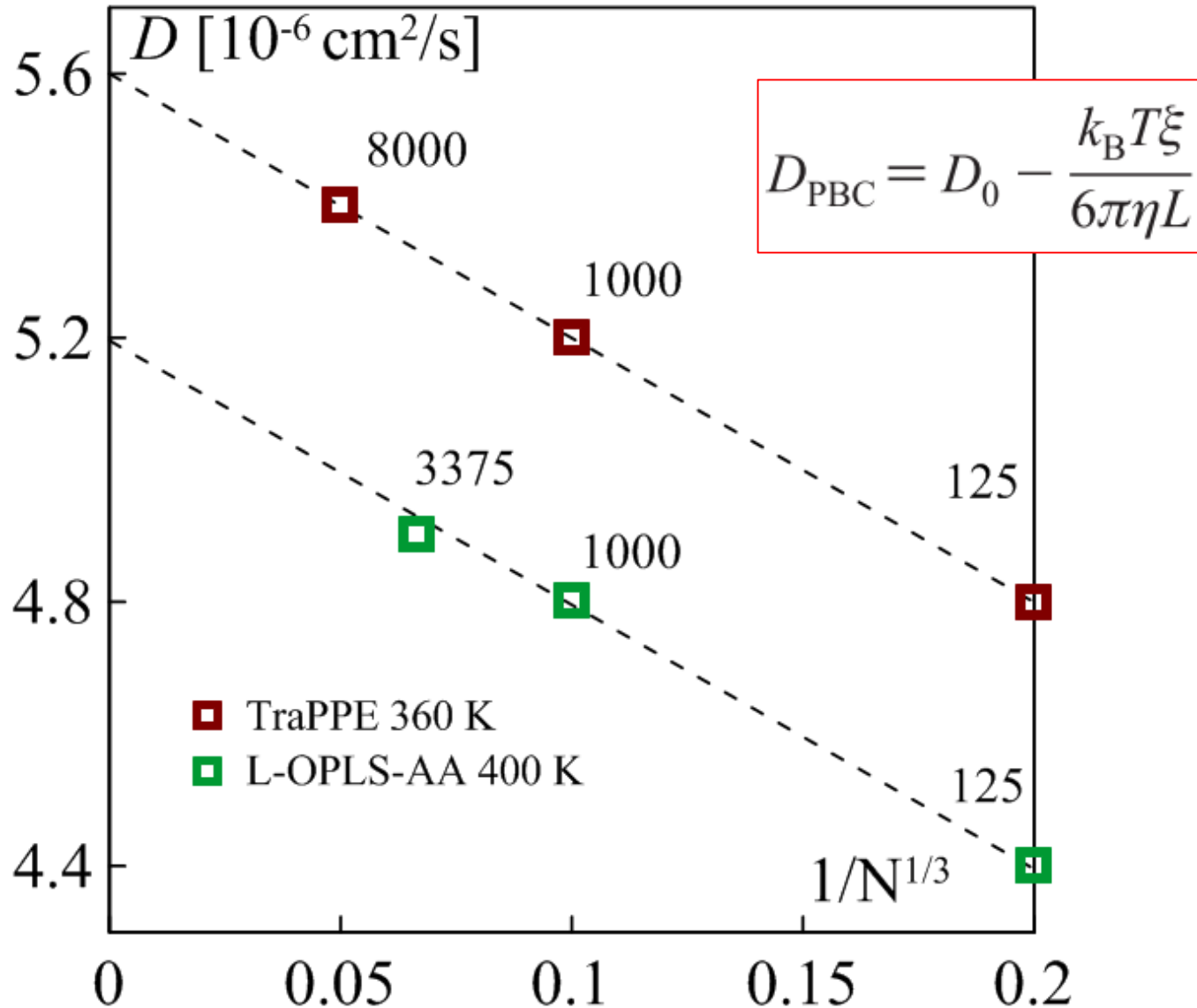
The 2<sup>nd</sup> stage of the averaging:

$$\langle \Delta r^2 \rangle = \sum_k^M \langle \Delta r^2 \rangle_{k\tau} / M$$

$M$  - Number of the zero shifts (~60)



# Размерные эффекты



# Техника усреднения

$$C_v(t) = \langle \vec{v}(0) \vec{v}(t) \rangle$$

The 1<sup>st</sup> stage of the averaging:

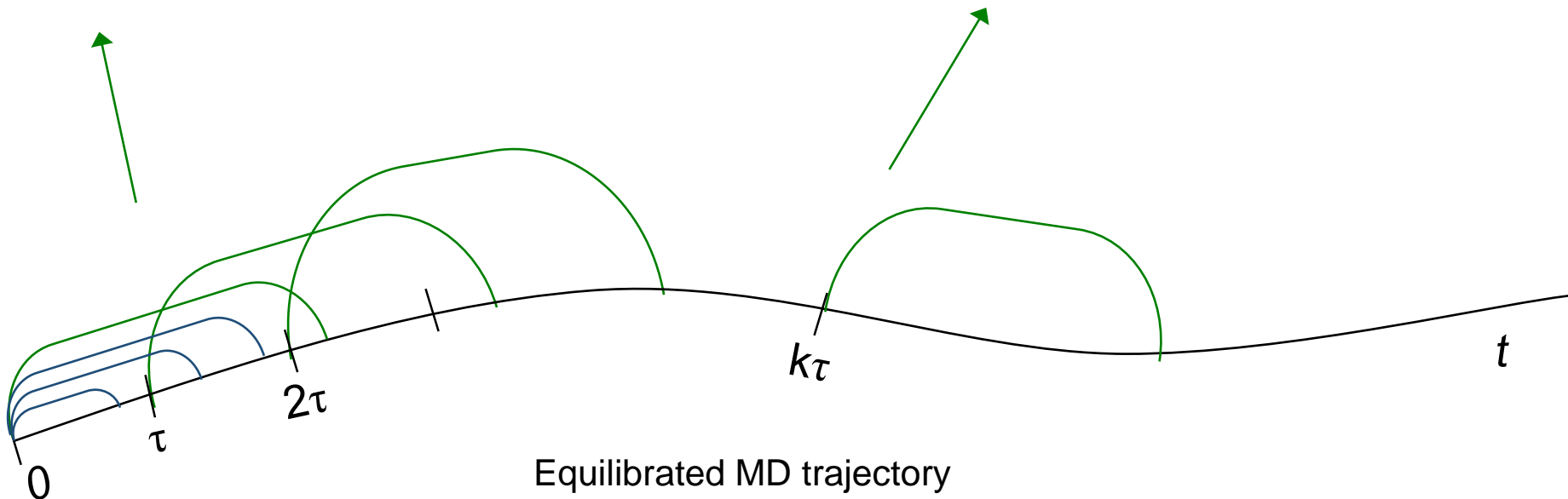
$$C_v(t) = \sum_N \vec{v}_i(0) \vec{v}_i(t) / N$$

$N$  - Number of the molecules (8000)

The 2<sup>nd</sup> stage of the averaging:

$$C_v(t) = \sum_k^M C_v(t)_{k\tau} / M$$

$M$  - Number of the zero shifts ( $\sim 300$ )



# План доклада

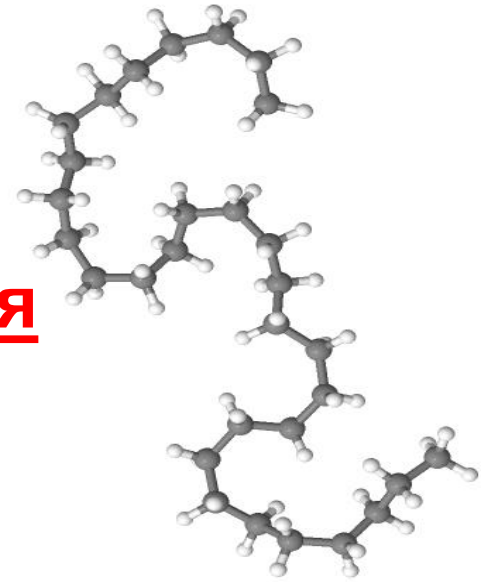


## 1. Самодиффузия в жидком n-триаконтане

### Потенциалы взаимодействия (+ COMPASS)

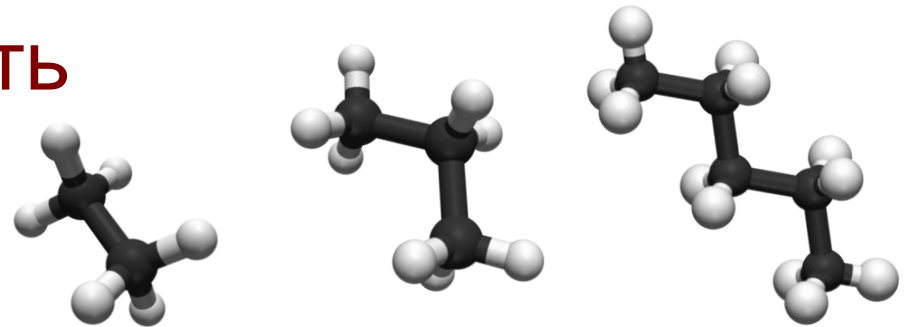
Сходимость Г-К

Предсказательная способность *Молекула n-триаконтана*



## 2. Сдвиговая вязкость

Сходимость Г-К



Неравновесная МД + эксперимент

# План доклада

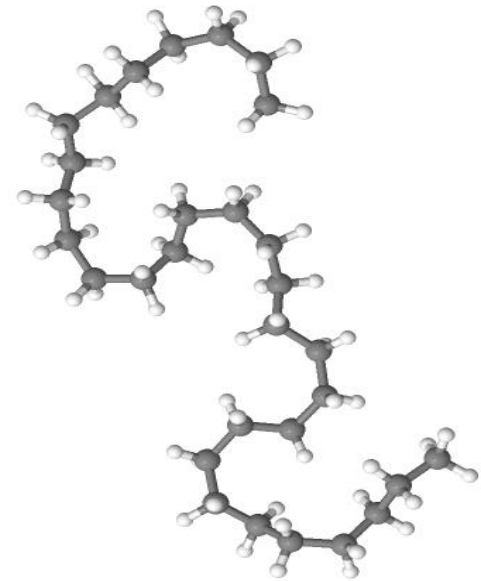


## 1. Самодиффузия в жидком *n*-триаконтане

Потенциалы взаимодействия (+ COMPASS)

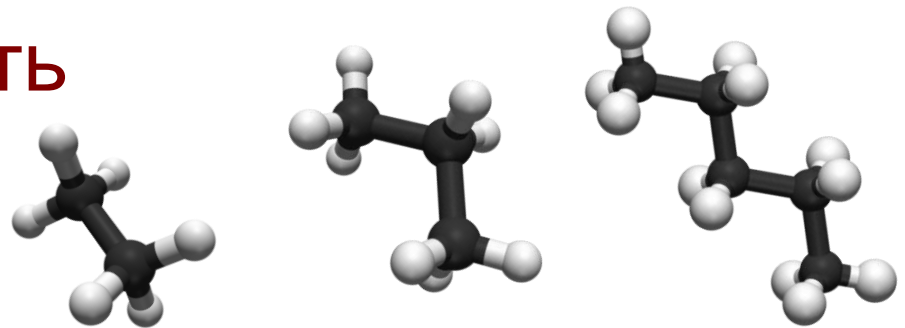
Сходимость Г-К

Предсказательная способность *Молекула *n*-триаконтана*



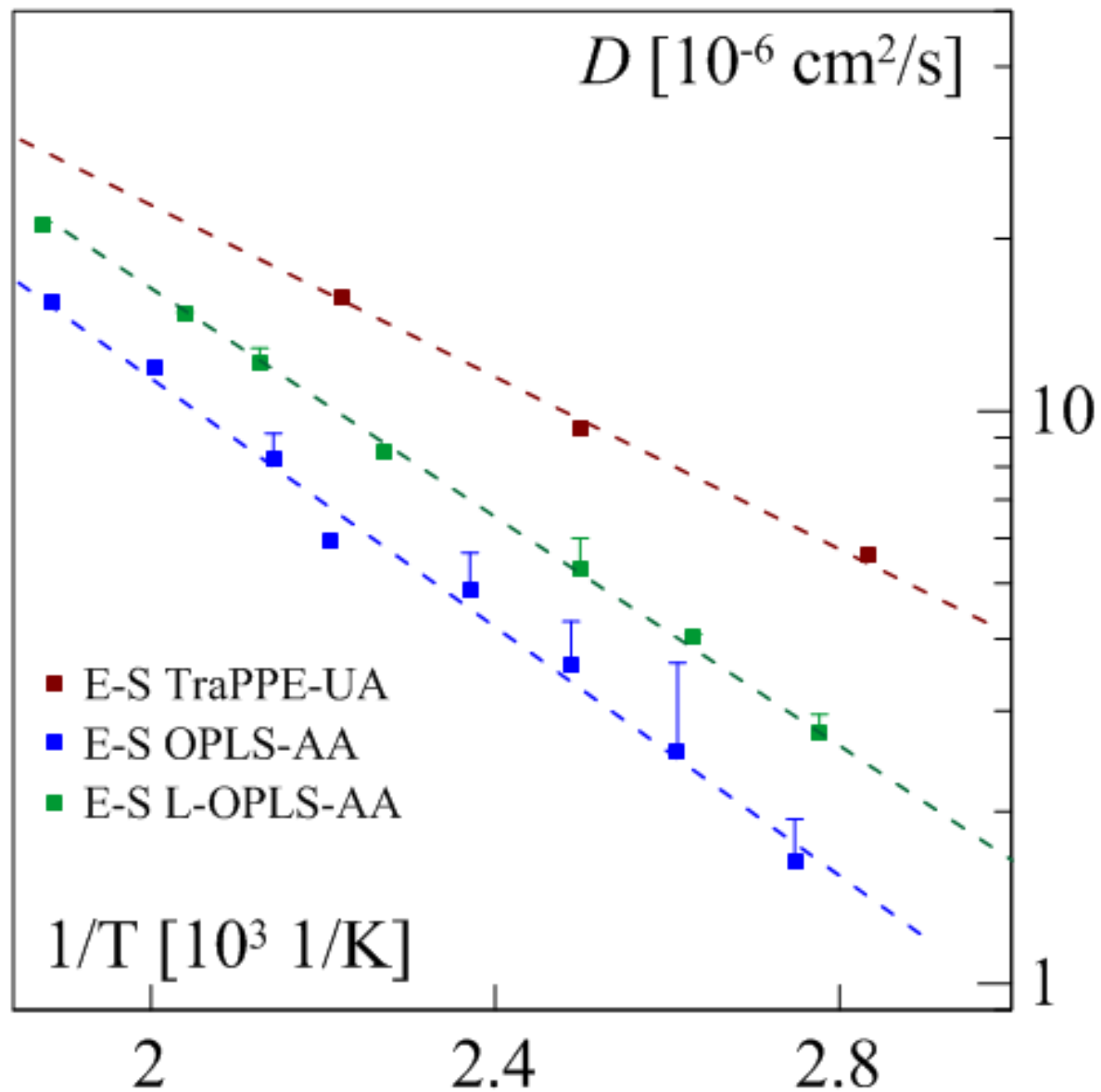
## 2. Сдвиговая вязкость

Сходимость Г-К



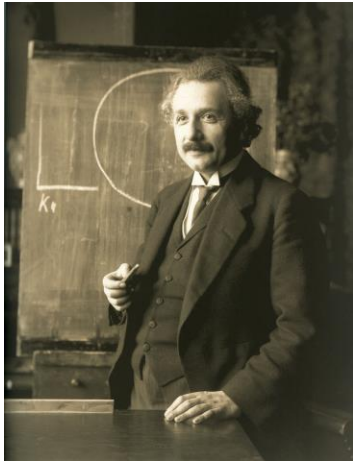
Неравновесная МД + эксперимент

# Результаты расчета методом Э-С





# Уравнения для D



Albert Einstein (1905)

They are  
theoretically  
**equivalent:**

$$\begin{aligned}
 D &= \lim_{t \rightarrow \infty} \frac{1}{2dt} \langle |x(t) - x(0)|^2 \rangle \\
 &= \frac{1}{2d} \lim_{t \rightarrow \infty} \frac{d}{dt} \langle |r(t) - r(0)|^2 \rangle \\
 &= \lim_{t \rightarrow \infty} \frac{1}{d} \langle (r(t) - r(0)) \cdot v(t) \rangle \\
 &= \lim_{t \rightarrow \infty} \frac{1}{d} \int_0^t d\tau \langle v(\tau) \cdot v(t) \rangle \\
 &= \lim_{t \rightarrow \infty} \frac{1}{d} \int_0^t d\tau \langle v(0) \cdot v(t - \tau) \rangle \\
 &= \frac{1}{d} \int_0^\infty dt \langle v(0) \cdot v(t) \rangle
 \end{aligned}$$



Melville S. Green (1954)



in Smoluchowski (1906)



Ryogo Kubo (1957)

# Сходимость в “простых” системах

$$D_{E-S} = D_{G-K}$$

S. H. Lee, D. K. Park, D. B Kang

Molecular Dynamics Simulations for Transport Coefficients of Liquid Argon : New Approaches,  
**Bull. Korean Chem. Soc. 2003. V. 24(2). P. 178.**

I. Gholami, A. Fiege, A. Zippelius

Slow dynamics and precursors of the glass transition in granular fluids.  
**Phys. Rev. E. 2011. V. 84. P. 1.**

M. Śmiechowski

Molecular hydrogen solvated in water – A computational study.  
**J. Chem. Phys. 2015. V. 143(24). P. 244505.**

# СХОДИМОСТЬ В “СЛОЖНЫХ” СИСТЕМАХ

J.P. Ryckaert, A. Bellemans

Molecular dynamics of liquid n-butane near its boiling point.

Chem. Phys. Lett. **1975**. V. 30(1). P. 123.

$$D_{E-S} < D_{G-K}$$

P. Padilla, S. Toxvaerd

Self-diffusion in n-alkane fluid models.

J. Chem. Phys. **1991**. 94(8). P. 5650.

P. Padilla, S. Toxvaerd.

Structure and dynamical behavior of fluid n-alkanes.

J. Chem. Phys. **1991**. V. 95(1). P. 509.

$D_{E-S}$  only

H. Lee, H. Pak Molecular dynamics simulation of liquid alkanes. II. Dynamic properties of normal alkanes: n-butane to n-heptadecane.

Bull. Kor. Chem. Soc. **1997**. V. 18(5). P. 478.

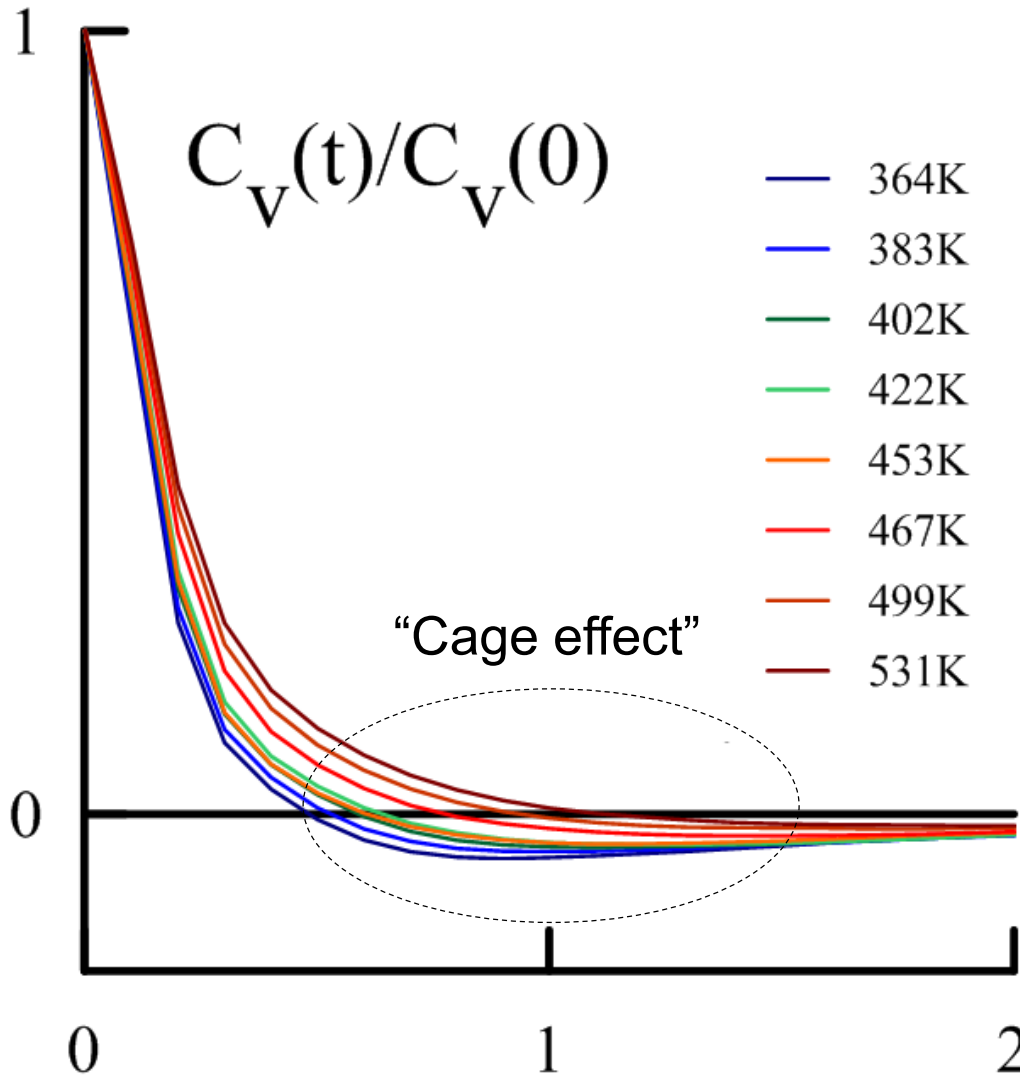
M. H. Kowsari, S. Alavi, M. Ashrafizaadeh, B. Najafi

Molecular dynamics simulation of imidazolium-based ionic liquids. I. Dynamics and diffusion coefficient. J. Chem. Phys. **2008**. V. 129(22). P. 224508.

H. Liu, E. Maginn, A. E. Visser, N. J. Bridges, E. B. Fox

Thermal and Transport Properties of Six Ionic Liquids: An Experimental and Molecular Dynamics Study. Ind. Eng. Chem. Res. **2012**. V. 51(21). P. 7242.

# Автокорреляторы скорости



“Cage effect”

$$N_{\text{rebounding}} > N_{\text{scattering}}$$

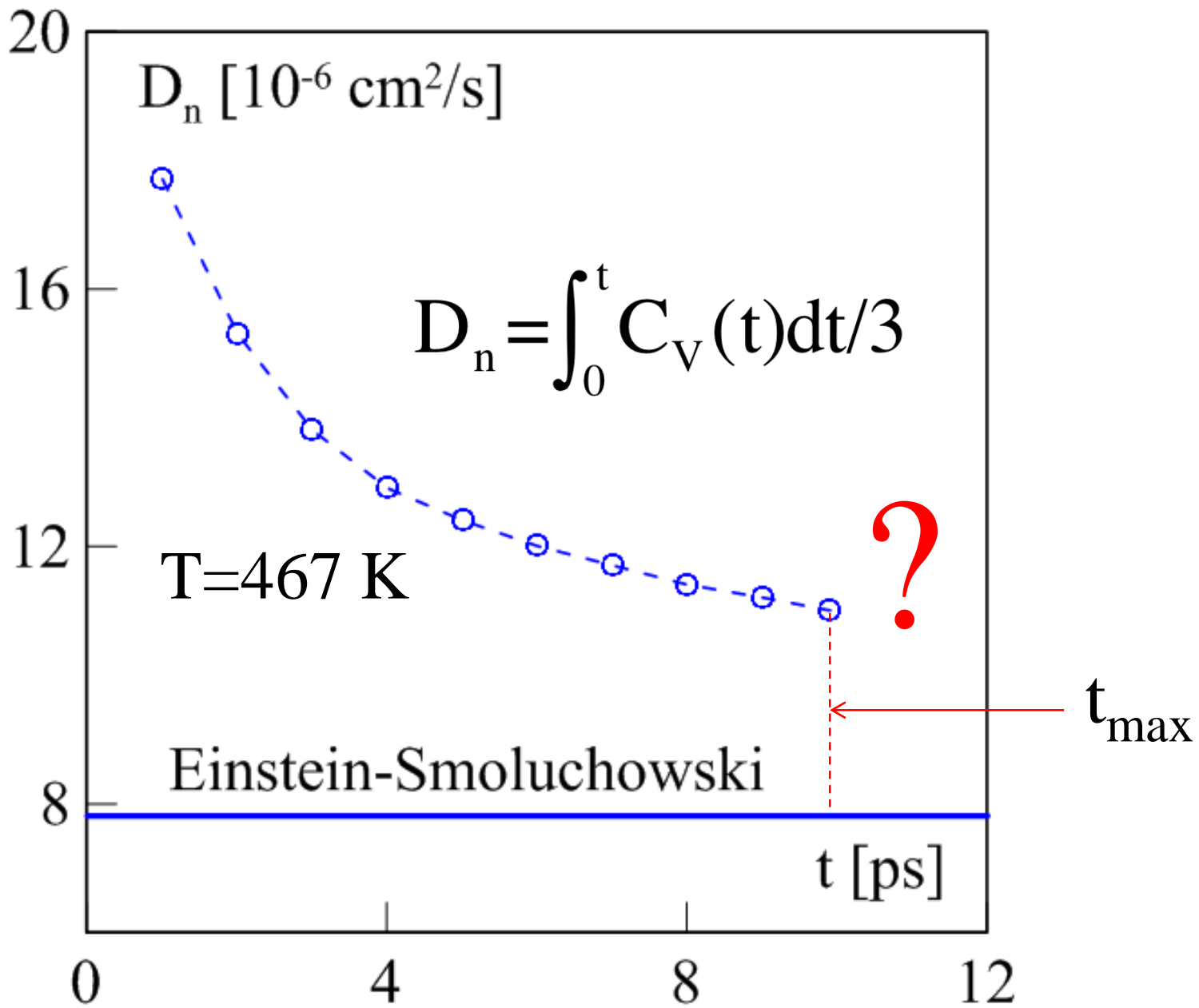
$$C_V(t) < 0$$

With the **increase** of T

$\rho$  ↘

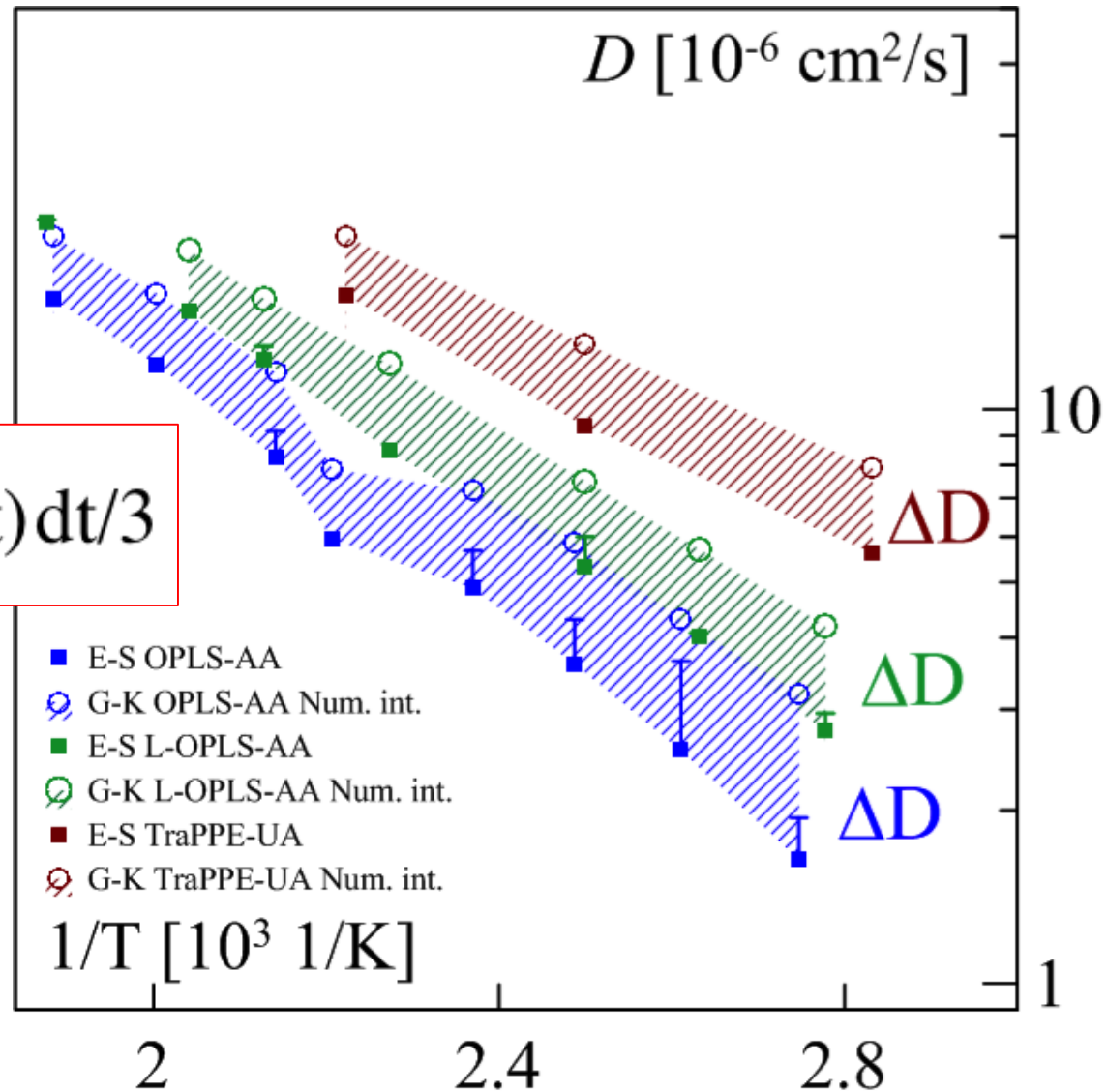
“Cage effect” **disappears**

# Результаты численного интегрирования



# Различие между Г-К и Э-С

$$D = \int_{t=0}^{7\text{ps}} C_V(t) dt / 3$$



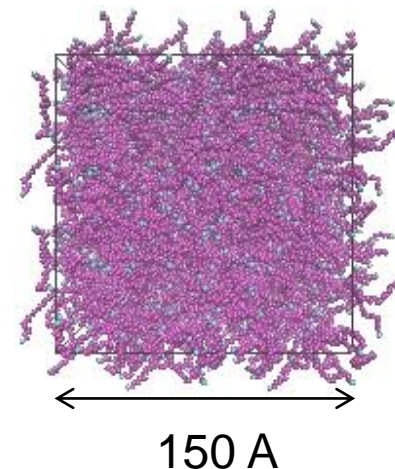
Значения Г-К **завышены**

# Правила интегрирования

## 1) Влияние периодических условий\*

$$t_{\max} \sim L / v_s \sim 150 \text{ \AA} / 1500 \text{ m/s} \sim 10 \text{ ps}$$

\*V. Ya. Rudyak, G. V. Kharlamov, and A. A. Belkin  
Diffusion of nanoparticles and macromolecules in dense gases  
and liquids // *High Temp.* **2001**. V. 39. No. 2. P. 264.



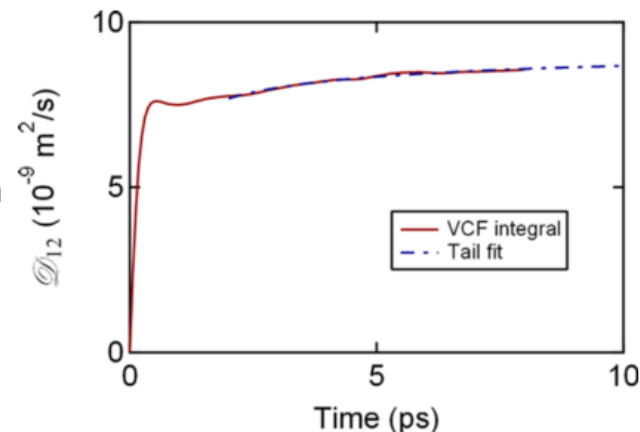
## 2) Момент, когда величина интеграла постоянна

I. Gholami, A. Fiege, A. Zippelius  
Slow dynamics and precursors of the glass transition in granular fluids.  
*Phys. Rev. E.* **2011**. V. 84. P. 1.

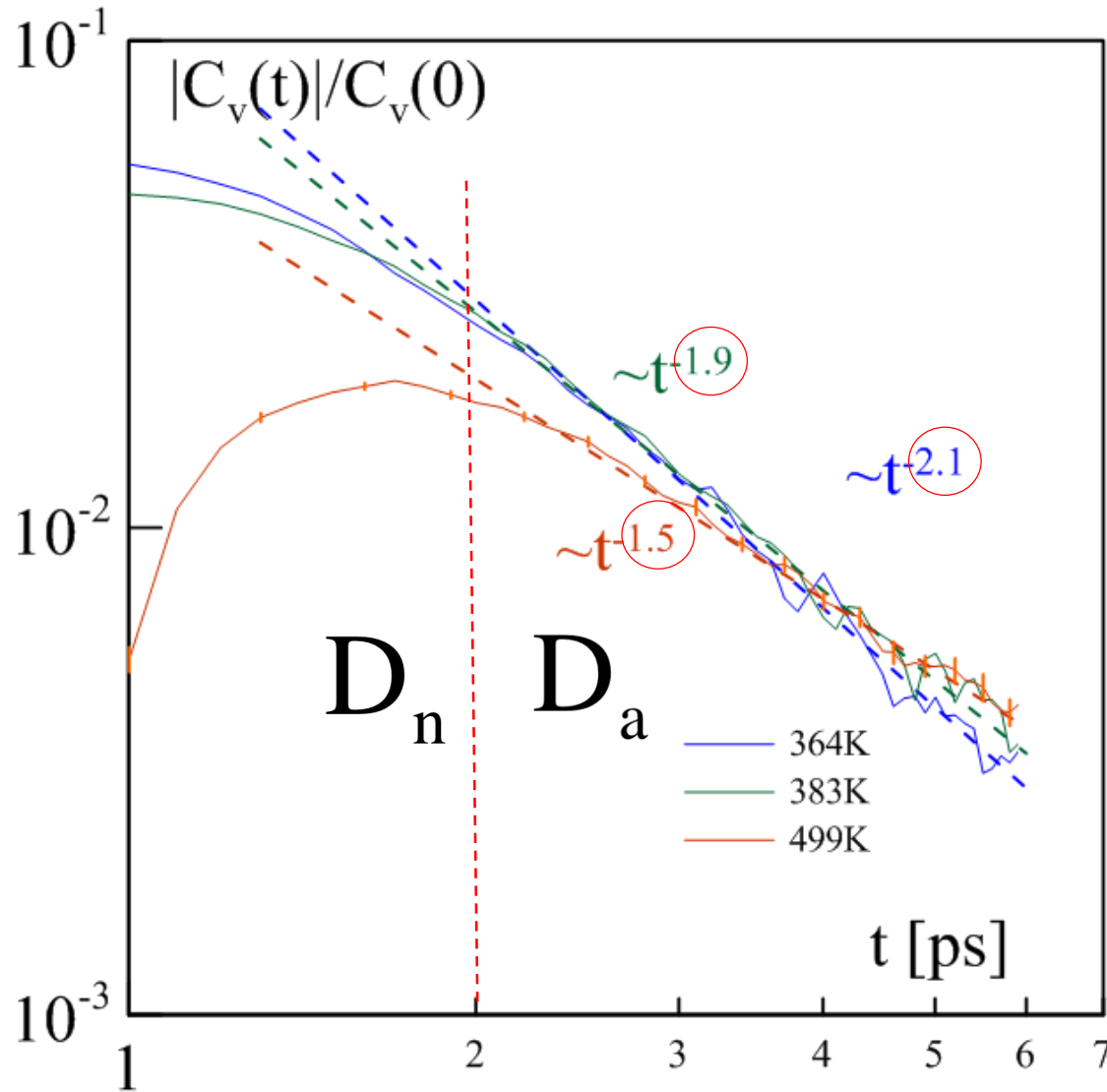
M. Śmiechowski  
Molecular hydrogen solvated in water – A computational study.  
*J. Chem. Phys.* **2015**. V. 143(24). P. 244505.

## 3) Найти асимптотику и продолжить интеграл

J. W. Nichols, D. R. Wheeler  
Fourier Correlation Method for Simulating Mutual Diffusion Coefficients in  
Condensed Systems at Equilibrium.  
*Ind. Eng. Chem. Res.* **2015**. V. 54(48). P. 12156.



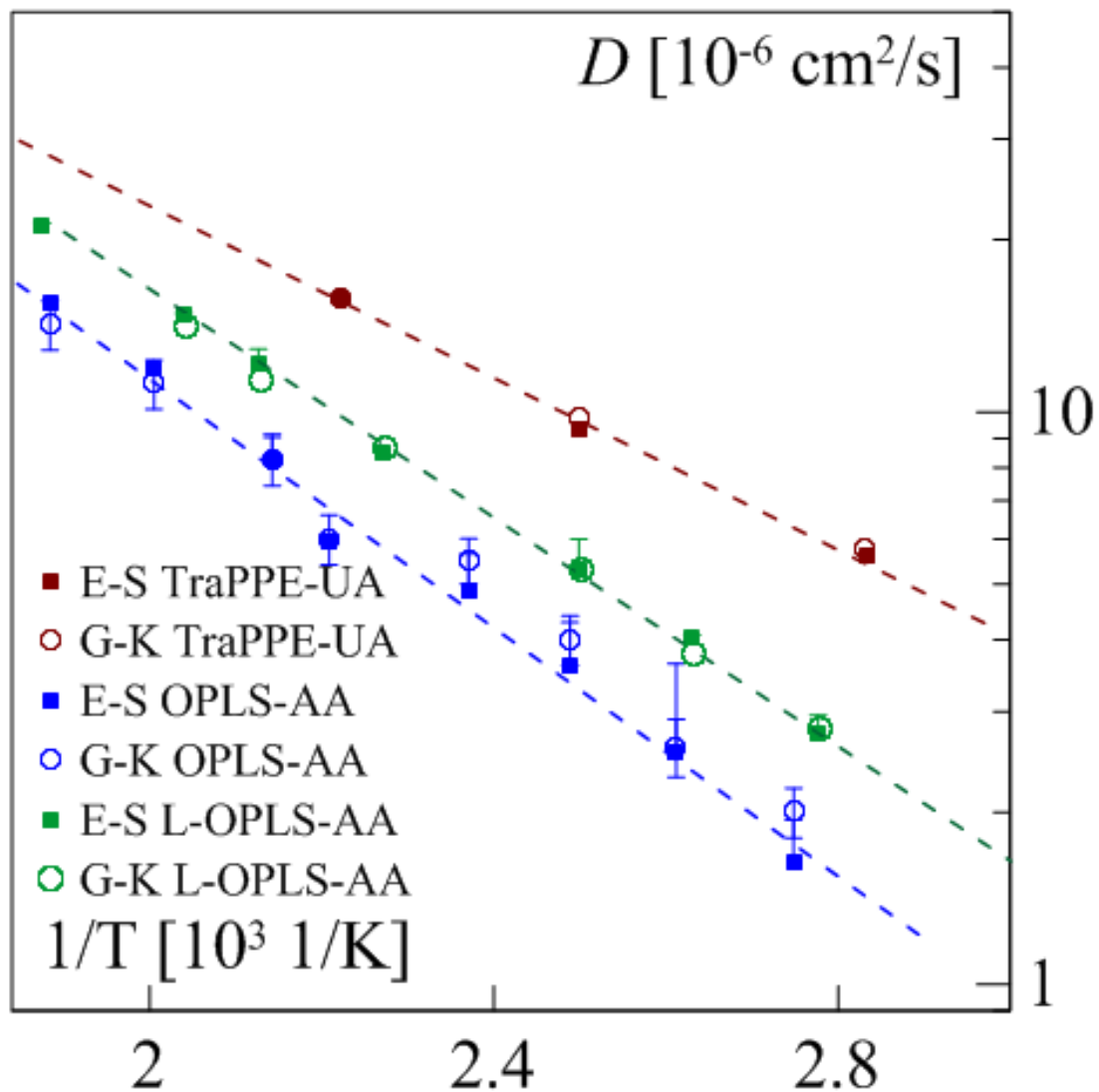
# Вклад асимптотики



$$\int_{t=0}^{2\text{ps}} C_v(t) dt / 3 \parallel D = D_n + D_a \parallel \int_{t=2\text{ps}}^{\infty} A t^{-\beta} dt$$



# Сходимость методов Г-К и Э-С



$$\sigma_{\beta} = 0.1$$

$$D_a^{\max} = \int_{t=2 \text{ ps}}^{\infty} A t^{-\beta_{\max}} dt / 3$$

$$D_a^{\min} = \int_{t=2 \text{ ps}}^{\infty} A t^{-\beta_{\min}} dt / 3$$

$$D_a^{\text{ave}} = \frac{[D_a^{\max}(\beta_{\max}) + D_a^{\min}(\beta_{\min})]}{2}$$

$$\sigma_D = D_a^{\max} - D_a^{\text{ave}}$$

# План доклада

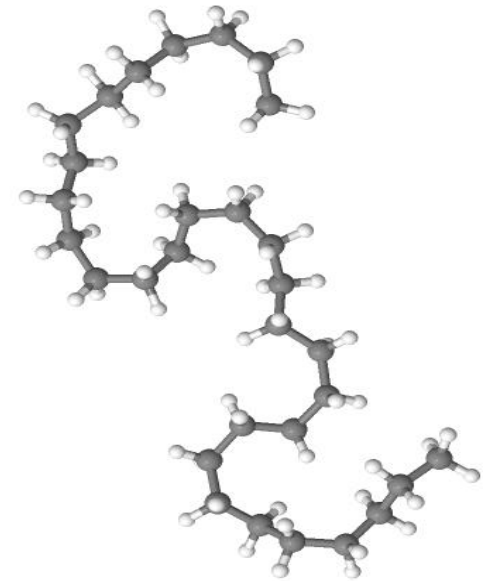


## 1. Самодиффузия в жидком *n*-триаконтане

Потенциалы взаимодействия (+ COMPASS)

Сходимость Г-К

Предсказательная способность

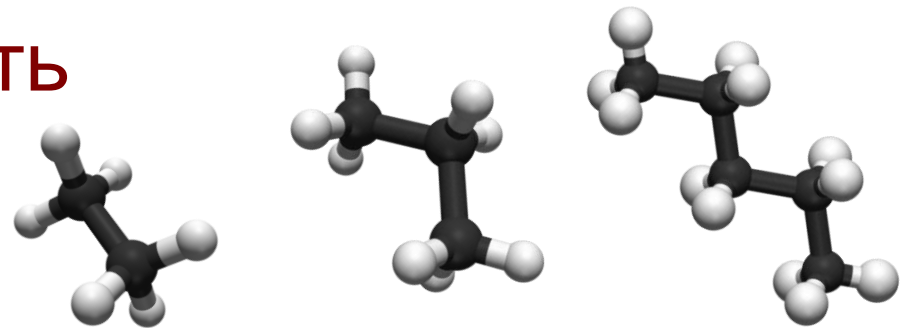


*Молекула *n*-триаконтана*

## 2. Сдвиговая вязкость

Сходимость Г-К

Неравновесная МД + эксперимент



# Выводы по части I

$$\int_{t=0}^{2\text{ps}} C_V(t) dt / 3$$

||

$$D = D_n + D_a$$

||<sub>-β</sub>

$$\int_{t=2\text{ps}}^{\infty} A t^{-\beta} dt$$

В системах с большими временами корреляции нельзя **численно** взять интеграл Грина-Кубо

Сходимость методов Грина-Кубо и Эйнштейна-Смолуховского с помощью **аналитического продолжения**

Модели класса I не воспроизводят **одновременно** УРС и D

У COMPASS почти получилось

